Spectral analysis of different harmonies
Implemented by
Equal temperament, Just, and Overtone ratio based tuning system

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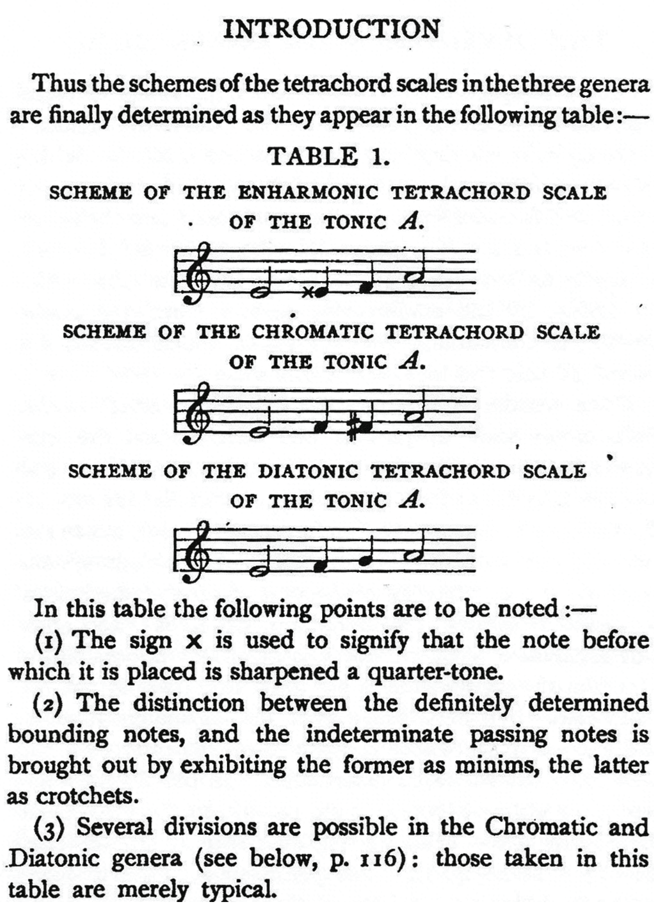
1. Introduction

Human started enjoying music from the pre-historic period. The consonances and dissonances, derived by small integer-related frequencies (consonances) or large integer-related frequencies (dissonances) were enjoyed by human for a long time and human gave their own order by “tuning” an instrument to systematically organize the sounds, which engendered the idea of “scale”. The earliest instrument, the bone flute invented by human about 35,000 years ago can play 7 notes scale[1], and this suggests that the history of tuning system and musical scale also started from the pre-historic era. As human civilization developed, the musical environment including instruments, musical scales, and tuning system also evolved. The tuning system and musical scales of Greek age also were significantly different from today’s equal temperament based tuning system and scales. For example, the Greek music what Aristoxenus suggests is entirely based on three fundamental musical units, including quarter-tones: less than a half-step in the equal-tempered musical system. (See FIGURE 1) Depending on the main musical context in a specific time-period, the musical tuning system was accustomed to the demands of musical milieu, and changed along with the evolution of music. The Equal-temperament tuning system widely used in modern music is the particularly customized tuning-system for the harmonic language established in the Baroque period. The harmony and counterpoint of 17th century was largely based on the concept of consonance and dissonance, in which the consonance includes octave (1:2), perfect 5th (2:3), perfect 4th (3:4), major 3rd (4:5), and minor 3rd (5:6), and the dissonance includes the rest of intervals. Although the equal-

[1] Steven Errede, Lecture note 8, p 10 http://courses.physics.illinois.edu/phys406/sp2014/Lecture_Notes/P406POM_Lecture_Notes/P406POM_Lect8.pdf, accessed [20140507, 00:37]

temperament tuning system took a quintessential role in the development of western classical music, now musicians are trying to enlarge the concept of consonance and dissonance, or even develop the theory of traditional harmonic language so to open up the possibilities of variegated expressions not yet discovered.


INTRODUCTION

Thus the schemes of the tetrachord scales in the three genera are finally determined as they appear in the following table:—

TABLE 1.

SCHEME OF THE ENHARMONIC TETRACHORD SCALE OF THE TONIC *A*.

SCHEME OF THE CHROMATIC TETRACHORD SCALE OF THE TONIC *A*.

SCHEME OF THE DIATONIC TETRACHORD SCALE OF THE TONIC *A*.

In this table the following points are to be noted:—

(1) The sign x is used to signify that the note before which it is placed is sharpened a quarter-tone.

(2) The distinction between the definitely determined bounding notes, and the indeterminate passing notes is brought out by exhibiting the former as minims, the latter as crotchets.

(3) Several divisions are possible in the Chromatic and Diatonic genera (see below, p. 116): those taken in this table are merely typical.

FIGURE1 - the scale units of Greek music[2]

For example, by using Just tuning system, one can listen the lost pure-ratio-related frequencies (4:5:6), which only can be expressed as 4:5.04:5.99 (A=440: C#=554.37: E=659.26) in the equal temperament system.

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[2] Henry Steward Macran, The Harmonics of Aristoxenus, Oxford at the Clarendon Press, p8

2. Consonance and Triads

The level of consonance and dissonance is not only related with the fundamental pitches vibrating simultaneously, but also with their overtone relations. The more of fundamental pitches are in smaller integer-related frequencies, the more of their overtones will be in the smaller integer-related harmonies. For example, the overtones of a fundamental and the overtones of an octave higher pitch should be all in the small integer-ratio related harmonies:

Fundamental	Overtone series in small integer relations								
f	f	2f	3f	4f	5f	6f	7f	8f	…
2f	2f	4f	6f	8f	10f	12f	14f	16f	…

FIGURE 2 - integer relations between a fundamental and it's an octave higher frequency

Even if two frequencies are not in a pure integer relation, human ear can regard those two frequencies as in consonance if their ratio deviations are less than a certain amount. This amount is named as just-noticeable difference, and a people psychophysically acknowledge the two pitches as the same if they differ by less than the just-noticeable difference.[3] (See FIGURE 3) If the difference from two different frequencies is less than the just-noticeable difference, human ear regards those two frequencies as same, or recognizes the difference as beating phenomena. In the latter case, one can hear just a single frequency slowly changing its amplitude, without noticing any frequency difference.

[3] Thomas D. Rossing, F. Richard Moore, Paul A. Wheeler, The Science of Sound 3rd Edition, Pearson Education, Inc., Addison Wesley, San Francisco, 2002, p. 123

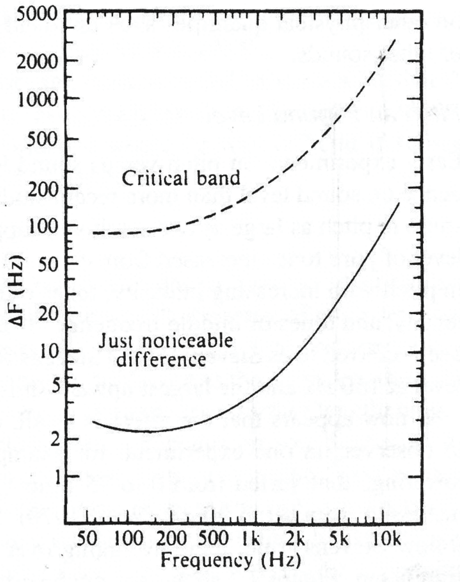

FIGURE 3 - Just-noticeable frequency difference[4]

For applying the just-noticeable frequency difference on the actual experiment, since it is impossible to apply the continuously changing Δf values on each harmonic series, the entire frequency range of the given graph is divided in five domains and the mean values Δf from each domain are applied for the frequencies in the range (FIGURE 4):

range	Δf
0-500Hz	3Hz
500-1000Hz	5Hz
1000-2000Hz	9Hz
2000-5000Hz	15Hz
5000Hz and up	30Hz

FIGURE 4 - quantized just noticeable frequency differences: Δf

4 Zwicker, E., G. Flottorp, and S. S. Stevens, Critical Bandwidth in Loudness Summation, J. Acoust. Soc. Am. 29: 548.

The consonance analysis of major and minor triad in Equal-temperament tuning system and just tuning system shows significant differences in their overtone relations when the just noticeable frequency difference Δf is applied:

Equal-tempered			Just-tuning		
freq	(5.04/4)	(5.99/4)	freq	(5/4)	(3/2)
110	138.59	164.81	110	137.5	165
220	277.18	329.62	220	275	330
330	415.77	494.43	330	412.5	495
440	554.36	659.24	440	550	660
550	692.95	824.05	550	687.5	825
660	831.54	988.86	660	825	990
770	970.13	1153.67	770	962.5	1155
880	1108.72	1318.48	880	1100	1320
990	1247.31	1483.29	990	1237.5	1485
1100	1385.9	1648.1	1100	1375	1650
1210	**1524.49**	**1812.91**	1210	1512.5	1815
1320	1663.08	1977.72	1320	1650	1980
1430	1801.67	2142.53	1430	1787.5	2145
1540	1940.26	2307.34	1540	1925	2310
1650	**2078.85**	2472.15	1650	2062.5	2475
1760	2217.44	2636.96	1760	2200	2640
1870	2356.03	2801.77	1870	2337.5	2805
1980	2494.62	2966.58	1980	2475	2970
2090	**2633.21**	3131.39	2090	2612.5	3135
2200	2771.8	3296.2	2200	2750	3300
2310	2910.39	**3461.01**	2310	2887.5	3465
2420	3048.98	3625.82	2420	3025	3630
2530	**3187.57**	3790.63	2530	3162.5	3795
2640	3326.16	3955.44	2640	3300	3960
2750	3464.75	4120.25	2750	3437.5	4125

2860	3603.34	4285.06	2860	3575	4290
2970	3741.93	4449.87	2970	3712.5	4455
3080	3880.52	4614.68	3080	3850	4620
3190	4019.11	4779.49	3190	3987.5	4785
3300	4157.7	4944.3	3300	4125	4950
3410	4296.29	5109.11	3410	4262.5	5115
3520	4434.88	5273.92	3520	4400	5280

FIGURE 5 - Major Triad - 4: 5.04: 5.99 in EQ temp and 4: 5: 6 in Just tuning

Equal-tempered			Just-tuning		
freq	(11.89/10)	(14.98/10)	freq	(6/5)	(3/2)
110	130.81	164.81	110	132	165
220	261.62	329.62	220	264	330
330	392.43	494.43	330	396	495
440	523.24	659.24	440	528	660
550	654.05	824.05	550	660	825
660	784.86	988.86	660	792	990
770	915.67	1153.67	770	924	1155
880	1046.48	1318.48	880	1056	1320
990	1177.29	**1483.29**	990	1188	1485
1100	1308.1	1648.1	1100	1320	1650
1210	**1438.91**	1812.91	1210	1452	1815
1320	1569.72	1977.72	1320	1584	1980
1430	1700.53	2142.53	1430	1716	2145
1540	1831.34	2307.34	1540	1848	2310
1650	1962.15	**2472.15**	1650	1980	2475
1760	**2092.96**	2636.96	1760	2112	2640
1870	2223.77	2801.77	1870	2244	2805
1980	2354.58	2966.58	1980	2376	2970
2090	2485.39	**3131.39**	2090	2508	3135
2200	2616.2	3296.2	2200	2640	3300
2310	**2747.01**	3461.01	2310	2772	3465
2420	**2877.82**	3625.82	2420	2904	3630

2530	3008.63	**3790.63**	2530	3036	3795
2640	3139.44	3955.44	2640	3168	3960
2750	**3270.25**	4120.25	2750	3300	4125
2860	3401.06	4285.06	2860	3432	4290
2970	3531.87	4449.87	2970	3564	4455
3080	3662.68	4614.68	3080	3696	4620
3190	3793.49	4779.49	3190	3828	4785
3300	3924.3	4944.3	3300	3960	4950
3410	4055.11	5109.11	3410	4092	5115
3520	4185.92	5273.92	3520	4224	5280

FIGURE 6 - Minor Triad - 10: 11.89: 14.98 in EQ temp and 10: 12: 15 in Just tuning

Here the yellow background frequencies have slightly larger Δf (mostly less than Δf * 2) than the noticeable differences in the specific range, and the red-lettered frequencies exist only in the equal tempered tuning. Although the Just tuned major and minor triads have significantly clearer frequency ratio relationships than the Equal tempered major and minor triads, many newly arisen integer related harmonies (yellow boxes and red-colored numbers) exist in Equal tempered tuning system in which the deviation is slightly larger than Δf and less than Δf * 2. In these frequencies, human ear can react differently on each frequency relation depending on the personal sensitivities and the level of musical trainings, but can be regarded as more consonant than the just tuning harmonies because of their numbers exceed those of just tuning sysem. This can be confirmed as more “stable” or “functional” harmony in human ear, when compared with the “pure” or “clear” qualities of just tuned harmonies, especially in major and minor triads, and pentatonic scale. However the graph does not show the actual amount of beatings which will be generated because of the Δf in Equal tempered system. The amount of beating also can be a quintessential factor for feeling a sound as consonant or dissonant, and this will be shown in the later experiment.

3. Just Tuning and Overtone Tuning, and Cents Deviations

Cents deviations between Equal temperament and Just tuning can be calculated by the following formula:

$$1 \text{ cent} = 2^{\frac{1}{1200}} = 1.000577$$

$$\text{EQ (Equal Tempered Freq)} \times (1.000577)^{x} = \text{JST (Just Freq)}$$

$$(1.000577)^{x} = \left(\frac{\text{JST}}{\text{EQ}}\right)$$

$$\text{X} \log(1.000577) = \log\left(\frac{\text{JST}}{\text{EQ}}\right)$$

$$\text{X} = \frac{\log \text{JST} - \log \text{EQ}}{\log(1.000577)}$$

FIGURE 7 - cents deviation formula

For the Just tuning ratios, I used following ratios:

Octave	Perfect 5th	Perfect 4th	Major 3rd	Minor 3rd
2/1	3/2	4/3	5/4	6/5

Major 2nd	Major 6th	Minor 7th	Augmented 4th	Minor 2nd
9/8	5/3	16/9	45/32	16/15

FIGURE 8 - Just tuning ratios

Here the Augmented 4th is generated from the perfect 4th (4/3) and the major 7th (15/8), and the major 7th (15/8) is generated from the major 3rd (5/4) and perfect 5th (3/2). Although this is not exactly same as the tuning system what Just-intonation related composers like Ben Johnston or Harry Partch used in their music, I wanted to limit the experiment within the clearest and smallest integer related ratios. For example, the other major 2nd ratio 10/9 to be used for making a just major 3rd is omitted here:

$$^{9}/_{8}\ \text{(just major 2nd)} \times {}^{10}/_{9}\ \text{(just major 2nd(2))} = {}^{5}/_{4}\ \text{(just major 3rd)}$$

Besides the Just tuning ratios, I also wanted to experiment the ratios based on the overtone tuning system. The overtone ratio-based tuning is intensively used by many modern classical music composers, including the Spectral school composers-Gérard Grisey and Tristan Murail, German composer Georg Friedrich Haas, and American composer Ben Johnston who also employed the overtone based ratios into his Extended Just Intonation System, in which he used ratios including 7/4, 12/7, 11/9, 16/11, and 18/11[5]. Haas also frequently used the overtone based tuning system in his music, for example in his piece *In Vain* he used various large number of higher partials of different fundamentals:

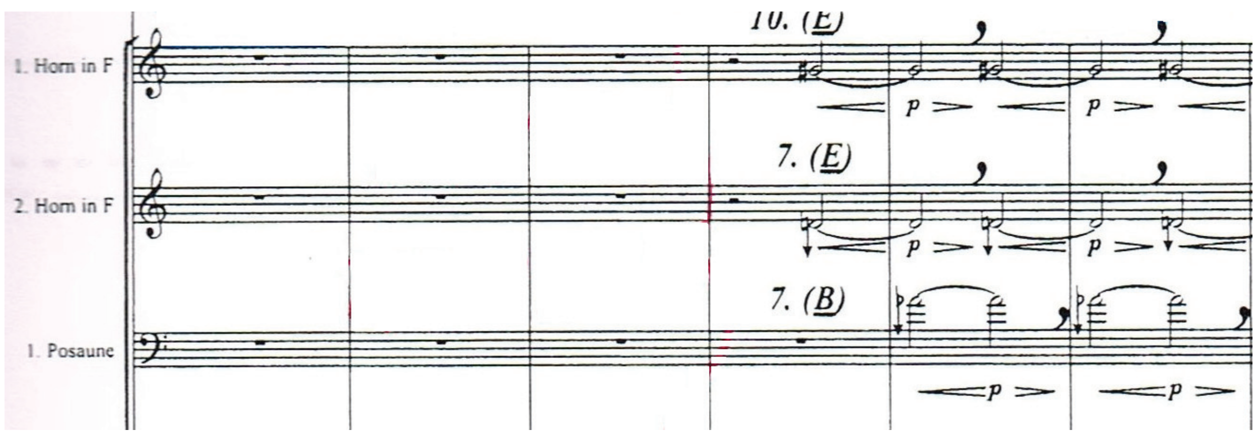

FIGURE 9 Hass, *In Vain,* m. 515[6]

In FIGURE 9, Hass indicated the actual overtone number on top of each note which should be tuned in the precise overtone based microtonal tuning. The fundamental is indicated in the

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5 Ben Johnston, edited by Bob Gilmore, Maximum Clarity, University of Illinois Press, Urbana and Chicago, p83-84

6 Georg Friedrich Haas, *In Vain*, Universal Edition, Vienna, 2000, p163 / m.515

parenthesis here.

For this tuning experiment I chose the ratios based on 7, 11, 13, and 17 which are rarely used for both in Equal temperament and Just tuning system, and have a distinctive quality. If these prime number overtone ratio based pitches are tuned in 24 equal tone tuning system, these pitches should be tuned a 1/4 tone lower than the other pitches:

Octave	Perfect 5th	Perfect 4th	Major 3rd	Minor 3rd
2/1	3/2	4/3	5/4	6/5

Major 2nd	Major 6th	Minor 7th	Augmented 4th	Minor 2nd
9/8	13/8	14/8	11/8	17/16

FIGURE 10   Overtone based tuning ratios

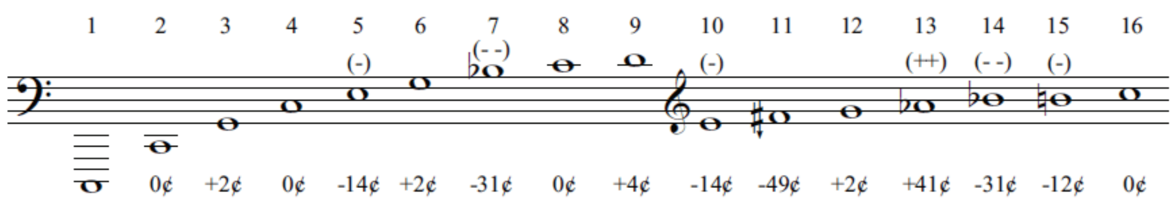

FIGURE 11   microtonal deviations in C overtone series (See the prime number overtones)[7]

For example, the 7th, 11th, and 13th harmonics' cents deviations are larger than 25 cents (See Figure 11), and are close to the quarter-tone tune downed pitches than the equal tempered pitches.

For selecting chords, eight crucial chords are selected from the classical musical context, modern

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[7] Robert Hasegawa, Eastman School of Music, Gegenstrebige Harmonik in the Music of Hans Zender, MTSNYS Annual Meeting, April 9, 2011

classical music, and non occidental music, based on the fundamental A2. The register is decided to attain maximum numbers of overtone series:

FIGURE 12 - eight fundamental chords from music history

These are major triad, minor triad, diminished triad, dominant seventh chord, diminished seventh chord, half-diminished seventh chord, *Petrushka* chord (what Stravinsky used in his piece *Petrushka* written in 1910-11), and pentatonic chord. The *Petrushka* chord is chosen both because it took a significant role of music historical context and also is based on order closed to the overtone series: 4:5:6:7:11:17.

4. Tuning Implementaion

Major triad	A	C-sharp	E				EQ
frequencies	110	138.59	164.81				
cents deviation	0	0	0				
	(1)	(5/4)	(3/2)				JS
frequencies	110	137.5	165				
cents deviation	0	-13.6886	1.997419				
Minor triad	A	C	E				EQ
frequencies	110	130.81	164.81				
cents deviation	0	0	0				
	(1)	(6/5)	(3/2)				JS
frequencies	110	132	165				
cents deviation	0	15.69956	1.997419				

Dimished triad	A	C	E-flat				EQ
frequencies	110	130.81	155.56				
cents deviation	0	0	0				
	(1)	(6/5)	(45/32)				JS
frequencies	110	130.81	154.69				
cents deviation	0	15.69956	-9.72273				
	(1)	(6/5)	(11/8)				OV
frequencies	110	130.81	151.25				
cents deviation	0	15.69956	-48.7097				
Dominant seventh	A	C-sharp	E	G			EQ
frequencies	110	138.59	164.81	196			
cents deviation	0	0	0	0			
	(1)	(5/4)	(3/2)	(16/9)			JS
frequencies	110	137.5	165	195.56			
cents deviation	0	-13.6886	1.997419	-3.89613			
	(1)	(5/4)	(3/2)	(14/8)			OV
frequencies	110	137.5	165	192.5			
cents deviation	0	-13.6886	1.997419	-31.2369			
Dimished seventh	A	C	E-flat	G-flat			EQ
frequencies	110	130.81	155.56	185			
cents deviation	0	0	0	0			
	(1)	(6/5)	(45/32)	(5/3)			JS
frequencies	110	130.81	154.69	183.33			
cents deviation	0	15.69956	-9.72273	-15.7203			
	(1)	(6/5)	(11/8)	(5/3)			OV
frequencies	110	130.81	151.25	183.33			
cents deviation	0	15.69956	-48.7097	-15.7203			

Half diminished seventh (= *TRISTAN Chord*)	A	C	E-flat	G			EQ
frequencies	110	130.81	155.56	196			
cents deviation	0	0	0	0			
	(1)	(6/5)	(45/32)	(16/9)			JS
frequencies	110	130.81	154.69	195.56			
cents deviation	0	15.69956	-9.72273	-3.89613			
	(1)	(6/5)	(11/8)	(14/8)			OV
frequencies	110	130.81	151.25	192.5			
cents deviation	0	15.69956	-48.7097	-31.2369			

Petrushka Chord (transposed)	A	C-sharp	E	B-flat	E-flat	G	EQ
frequencies	110	138.59	164.81	233.08	311.13	392	
cent deviation	0	0	0	0	0	0	
	(1)	(5/4)	(3/2)	(32/15)	(45/16)	(32/9)	JS
frequencies	110	137.5	165	234.67	309.38	391.11	
cents deviation	0	-13.6886	1.997419	11.78595	-9.77845	-3.94046	
	(1)	(5/4)	(3/2)	(34/16)	(22/8)	(28/8)	OV
frequencies	110	137.5	165	233.75	302.5	385	
cents deviation	0	-13.6886	1.997419	4.976177	-48.7655	-31.2369	

Pentatonic Chord	A	B	C-sharp	E	F-sharp		EQ
frequencies	110	123.47	138.59	164.81	196		
cent deviation	0	0	0	0	0		
	(1)	(9/8)	(5/4)	(3/2)	(5/3)		JS
frequencies	110	123.75	137.5	165	183.33		
cents deviation	0	3.926938	-13.6886	1.997419	-15.7203		
	(1)	(9/8)	(5/4)	(3/2)	(13/8)		OV

frequencies	110	123.75	137.5	165	178.75		
cents deviation	0	3.926938	-13.6886	1.997419	-43.8595		

5. Results

In dominant seventh chord, there were clear difference between Equal temperament and Overtone based tuning.

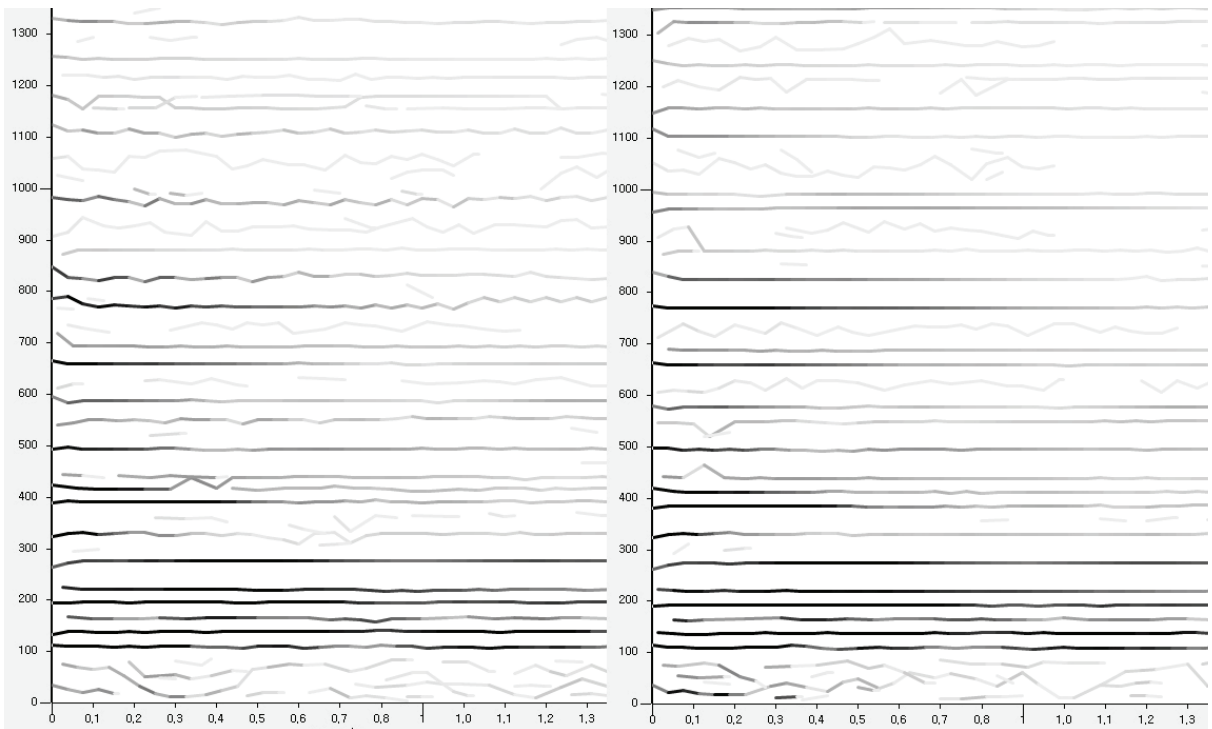

FIGURE 13 dominant seventh chord in EQ (Equal Temperament) and OVT (Overtone) tuning

While the higher overtones from Equal temperament dominant seventh chord shows an unstable quality caused by beating phenomenon, the Just tuned dominant seventh chord's spectrum is much more stabilized.

The largest difference occurred at the 3rd harmonics of the fundamental-A2 (here E4), which coincide with the 2nd harmonics of E3, which is also E4:

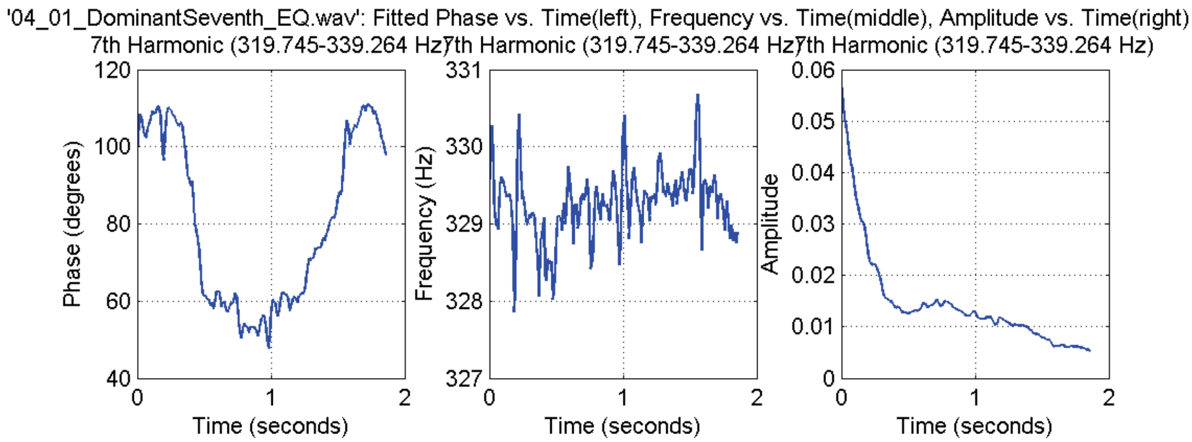

FIGURE 14 – 3rd harmonics of dominant seventh harmony tuned in EQ

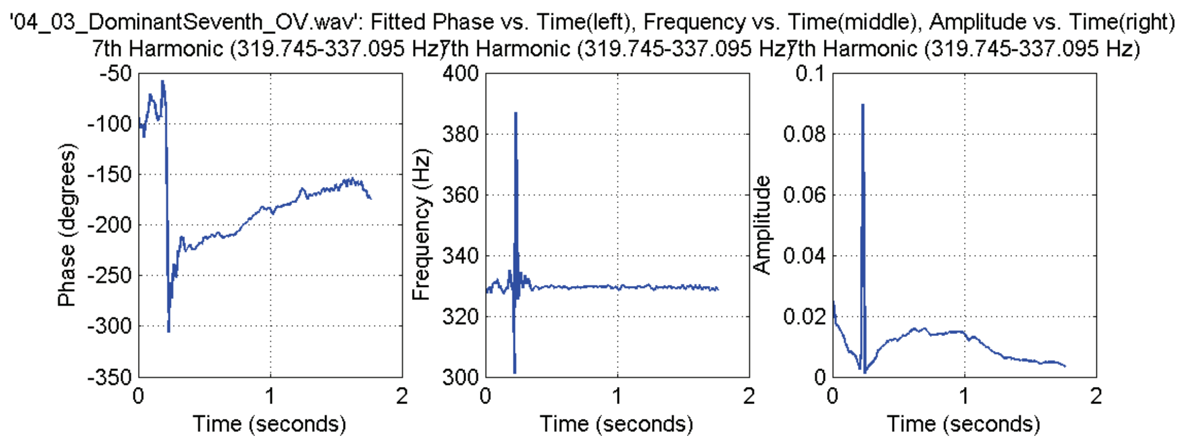

FIGURE 15 – 3rd harmonics of dominant seventh harmony tuned in JST and OVT

The harmonic fluctuation of E3 of Equal temperament system comes from the frequency beating phenomenon, between the third harmonics of A2=330 Hz and the second harmonics of E3=329.62 Hz. Beating phenomenon comes from the combination of two complex signals:

$$Z_1(\vec{r},t) = A_1(\vec{r},t)e^{i(w_1 t + \varphi_1)} = A_1(\vec{r},t)(\cos w_1 t + i \sin w_1 t)$$

$$Z_2(\vec{r},t) = A_2(\vec{r},t)e^{i(w_2 t + \varphi_2)} = A_2(\vec{r},t)(\cos w_2 t + i \sin w_2 t)$$

The total amplitude becomes:

$$|Z_{total}| = |Z_1|^2 + |Z_2|^2 + 2\mathrm{Re}\{Z_1 Z_2^*\}$$

$$= A_1(\vec{r},t)^2 + A_2(\vec{r},t)^2 + 2\mathrm{Re}\left\{A_1(\vec{r},t)A_2(\vec{r},t)e^{i\{(w_1 t + \varphi_1) - (w_2 t + \varphi_2)\}}\right\}$$

If we change $e^{i\{(w_1 t + \varphi_1) - (w_2 t + \varphi_2)\}}$ into trigonometry function, this becomes:

$$|Z_{total}| = A_1(\vec{r},t)^2 + A_2(\vec{r},t)^2 + 2A_1(\vec{r},t)A_2(\vec{r},t)\cos[(w_1 - w_2)t + (\varphi_1 - \varphi_2)]$$ [8]

By using this formula, we can see the beating in 0.38 Hz comes out from the frequency difference between 330 Hz and 329.62 Hz, since 0.38 Hz is less than the *critical band* for human hearing. This beating only comes out from the equal tempered dominant chord, and is not prominent at the Just tuning and Overtone tuning system, which is demonstrated in the result above. This phenomenon becomes more prominent in the higher frequencies in Equal temperament system, since the amounts of frequency deviations become larger in the higher overtone area as FIGURE 13 shows. For example, the 10th overtone of A2 is 1100 Hz, and the 8th overtone of C#3 is 1108.72 Hz, and the beating frequency here is 8.72 Hz which is much larger than the lower harmonics. There will be much more beating frequencies in the higher overtones, since the integer multiples of Least Common Multiple of the three fundamentals will coincide in the higher register. The difference caused by beating becomes clearer in the pentatonic chord because of the pure integer ratios and more fundamentals, and we will see later. Also, the relative phase shift of each harmonic was much more adjacent in the OVT tuning:

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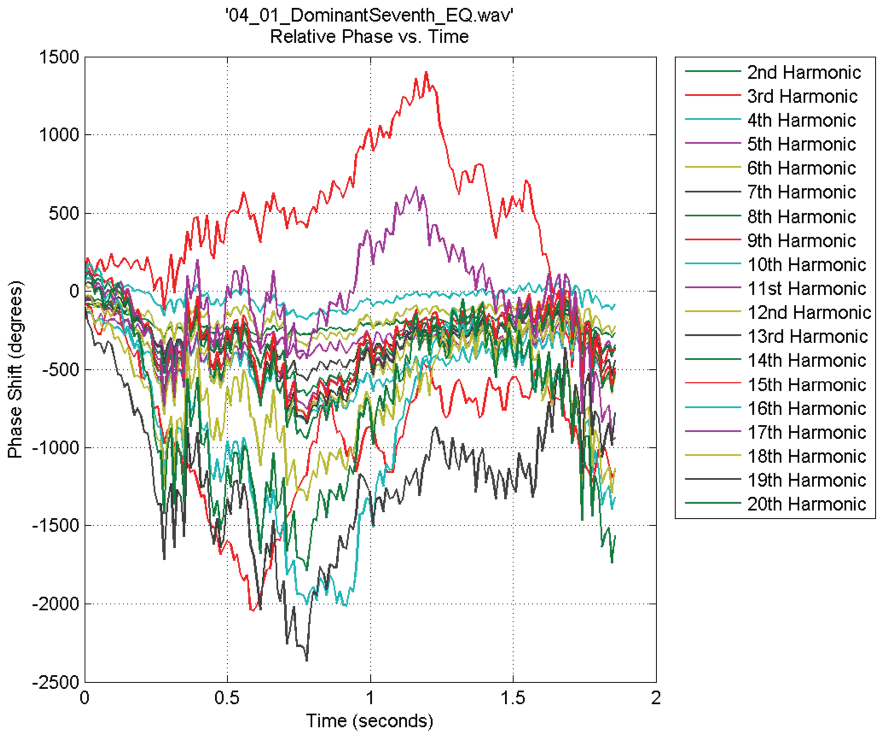
[8] Errede, Lecture note 11, p 7-10, http://courses.physics.illinois.edu/phys406/sp2014/Lecture_Notes/P406POM_Lecture_Notes/P406POM_Lect11.pdf, accessed [20140507, 00:39]



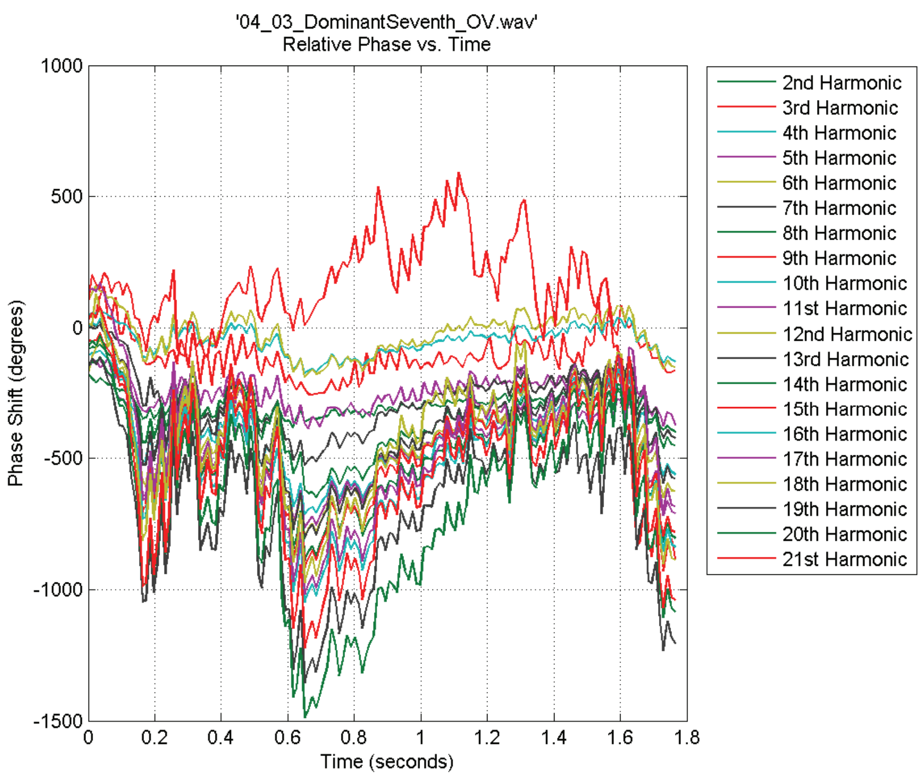


FIGURE 16 – relative phase shift of dominant seventh chord in EQ and OVT tuning

In a chord like *Petrushka* chord, the OVT tuned chord spectrum shows remarkably abundant higher frequencies than the EQ tuned chord, especially from 800 Hz to 1000 Hz:

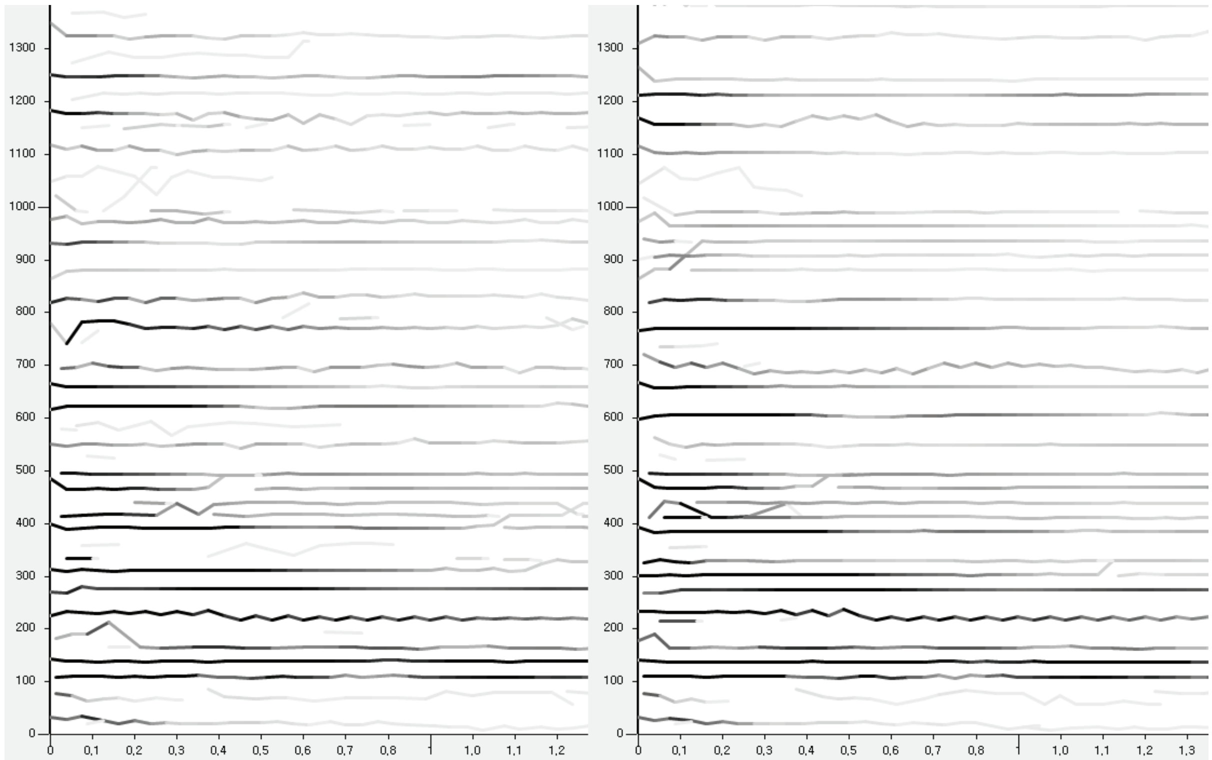
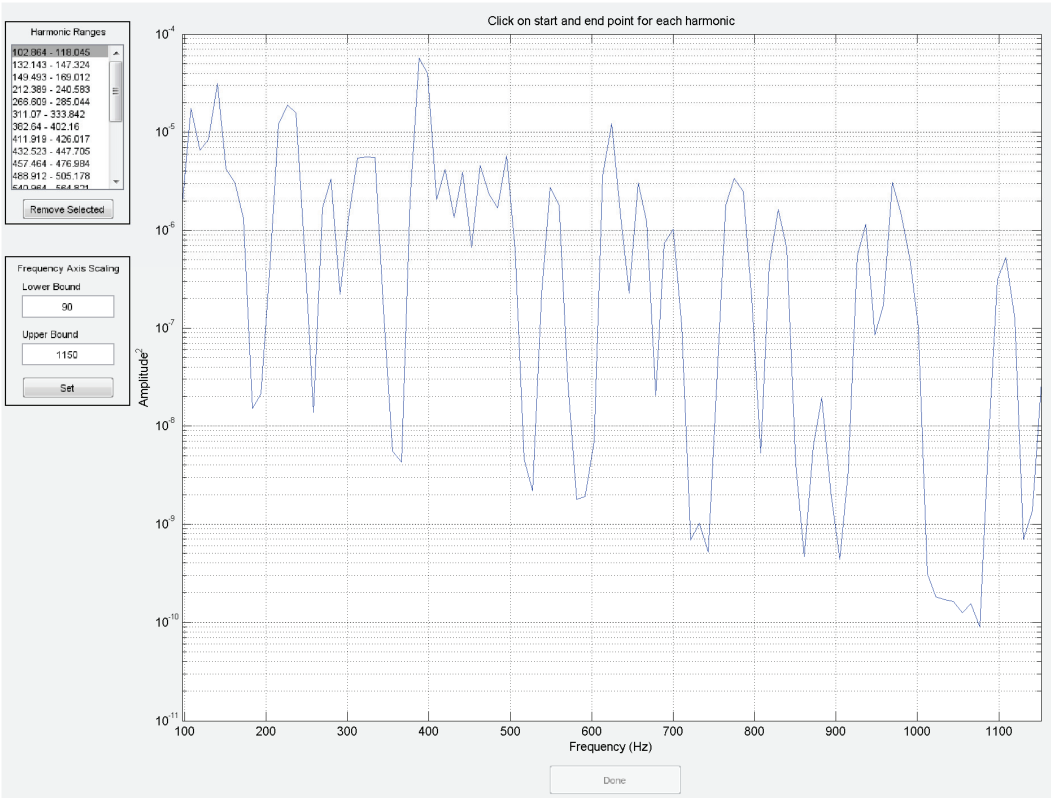


FIGURE 17 – *Petrushka* chord spectrum in EQ and OVT tuning

Not only the OVT tuned chord spectrum shows rich harmonic spectrum, but also the higher overtones are in more stable qualities than those of EQ tempered chord harmonics. This can be identifiable in FIGURE 18, in which the peak of each harmonic partial is more distinguishable in the Overtone tuning than the Equal temperament tuning especially between 200 Hz and 500 Hz. (See FIGURE 18) This can open up the new ways of interpreting modern classical compositions which employ significantly larger amount of dissonances than the previous classical compositions, and give a new direction for the systematical compositional approaches based on the Acoustic phenomena in the classical music of future.



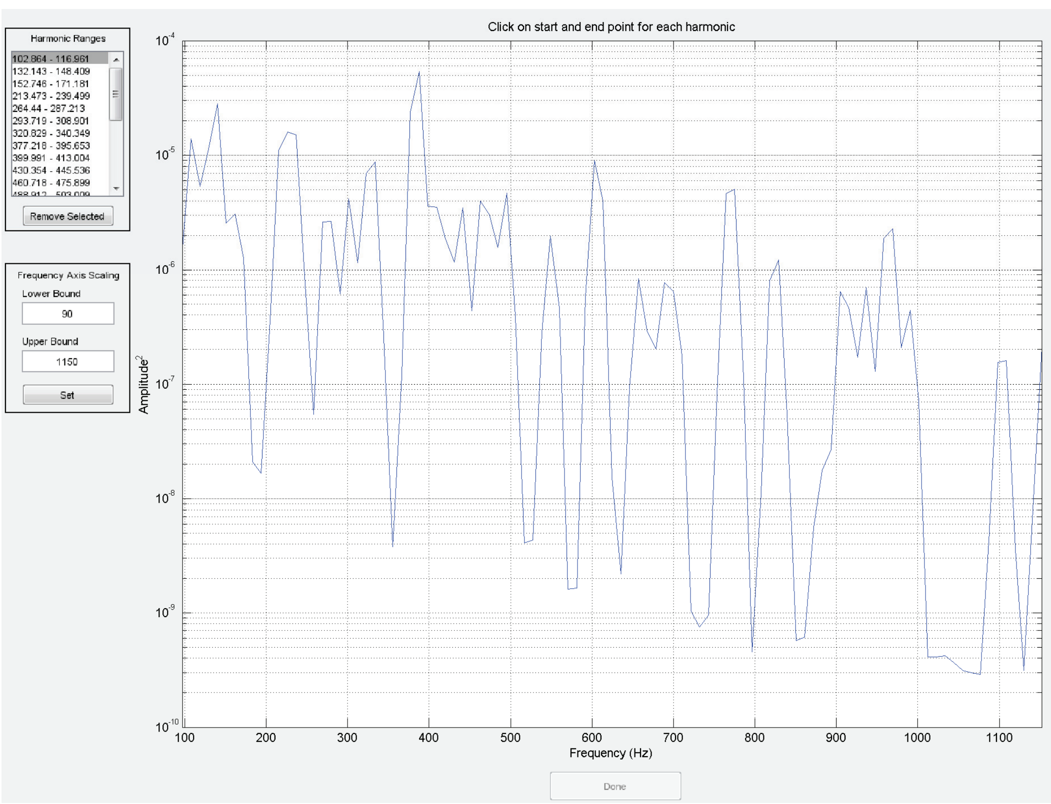


FIGURE 18 – *Petrushka* chord spectrum in EQ and OVT tuning

Another striking difference came out from the pentatonic chord:

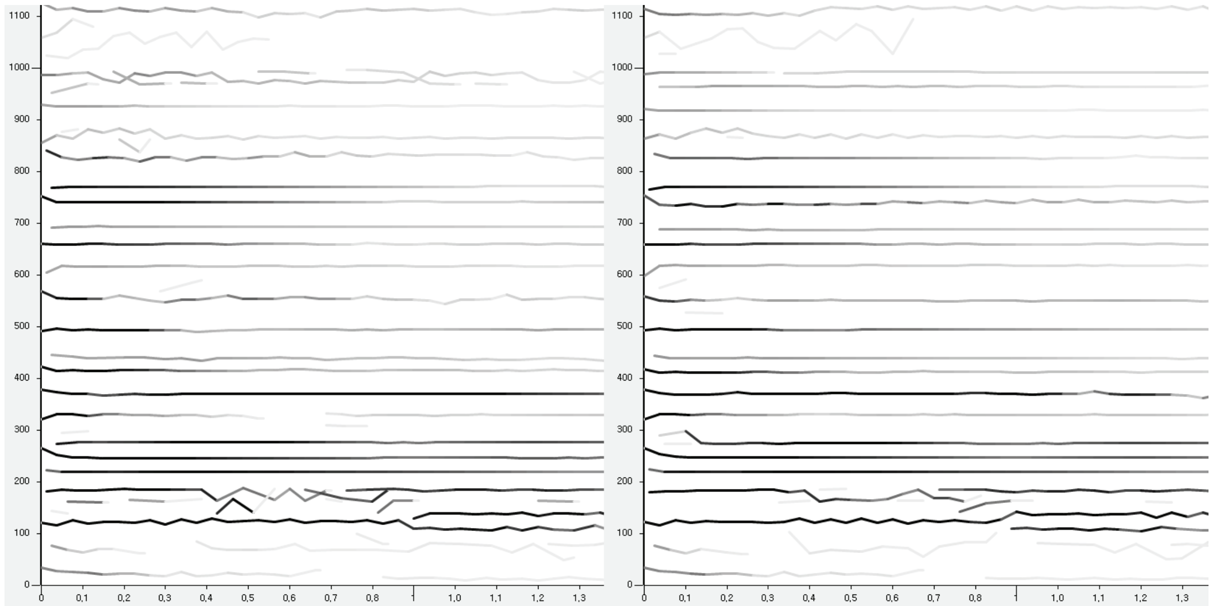


FIGURE 19 – pentatonic chord in EQ and JST tuning

This is not easily identifiable from the spectrogram above, but easily distinguishable by ear. As the spectrogram suggests, the chordal quality of pentatonic chord is prominent in the 2nd and 3rd harmonics, while the 1st harmonics (fundamentals) are not prominent and steady as shown above. (See FIGURE 19, where the frequencies below 196 are not prominent and unstable.) However, from the 2nd harmonics, the frequencies show clearly identifiable amplitudes, and the clear integer ratios between the pitches become more emphasized. For example, the 3rd harmonics of EQ tuned pentatonic chords are in the ratio of 3:3.37:3.78:4.49:5.35 = **24:26.96:30.24:35.92:42.8** while the JST tuned pentatonic chords are in the ratio of 3:3.375:3.75:4.5:5 = **24:27:30:36:40** which are in pure integer ratio relationships. The difference between these two ratios is quite distinctive when one listens to the 3rd harmonics only, even to those who is not familiar with western musical system. For example, the fourth harmonics of low B(123.47 Hz) is 493.88 Hz, and this meets with the third harmonics of E

(164.81 Hz) = 494.43. These harmonics generate beating in the Equal temperament system in 0.55 Hz. This beating is much lesser in the just tuned chord, in which the fourth harmonics of low B (123.75 Hz) and the third harmonics of E (165 Hz) are exactly identical = 495 Hz, if the pitches are tuned precisely. (See Figure 20, 21)

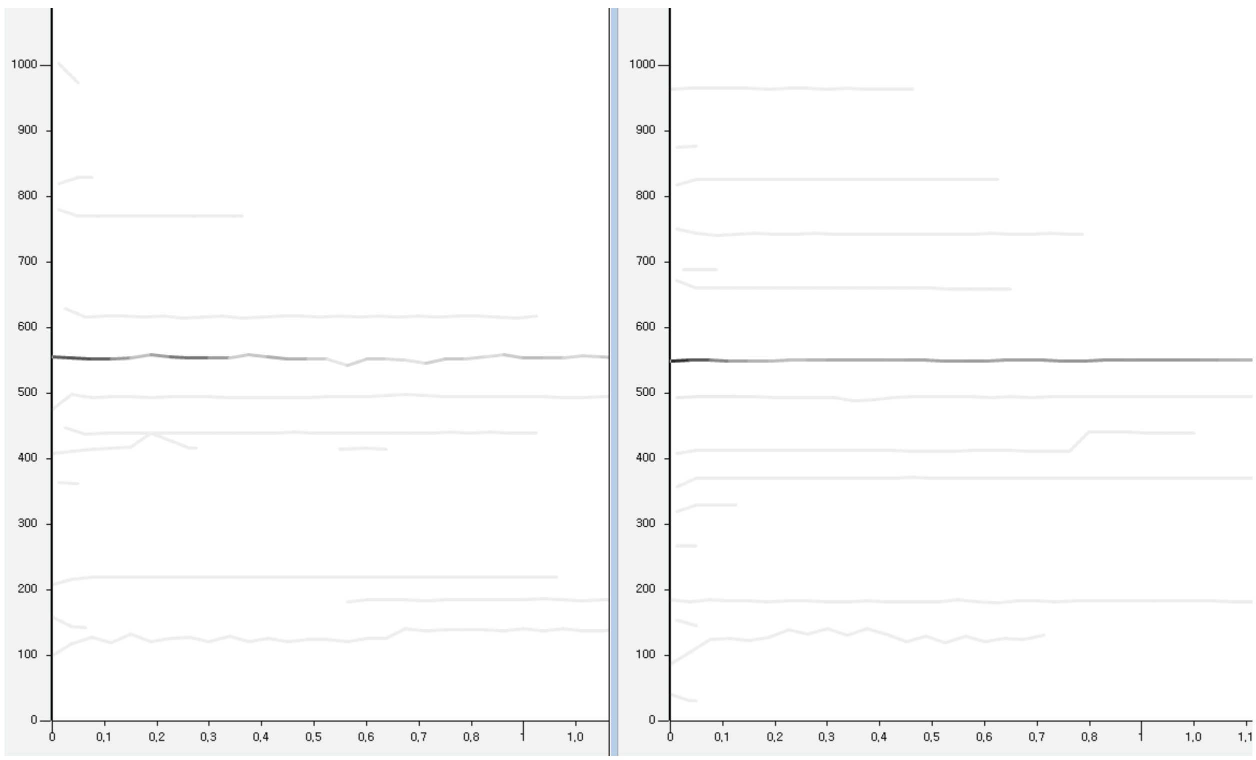


FIGURE 20 – the 4th harmonics of B and 3rd harmonics of E from EQ and JST tuning

The degrees of phase shift from each harmonics also are more consistent in the Just tuned pentatonic chord. (See Figure 22) If the beating phenomena are added which is aforementioned in the dominant seventh chord, the difference becomes clearer.

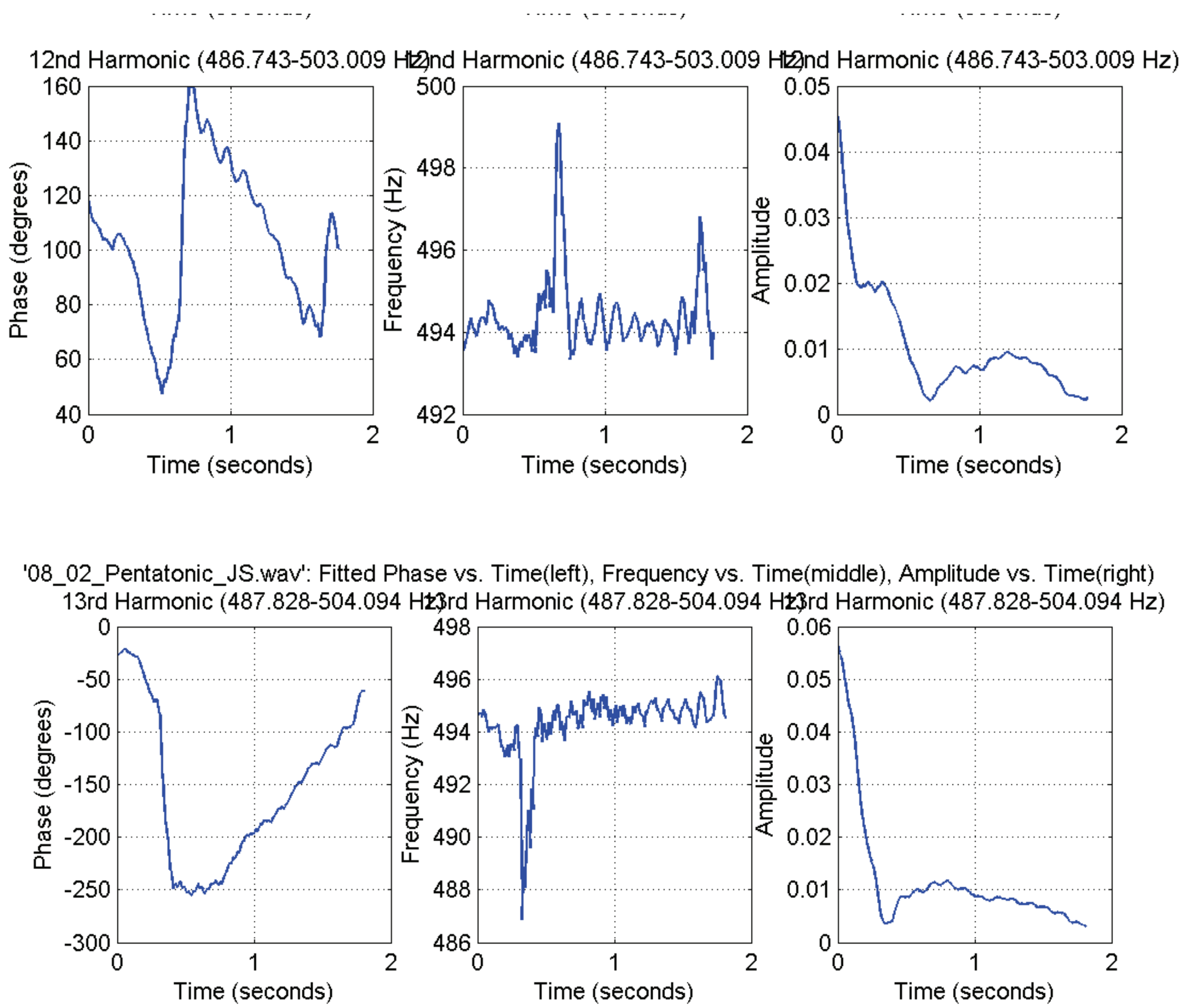


FIGURE 21 – the 4th harmonics (495 Hz) from pentatonic chord in EQ and JST tuning

This can demonstrate the reason why the East Asian musical scales based on the pentatonic system do not use Equal temperament tuning system; the pentatonic scale sounds much more stable and balanced when it is based on Just tuning system. This feature is also shown in the FIGURE 22 and 23-spectrograms of both tunings. While the overtones from 330 Hz to 3300 Hz of Just tuned chord are more stable without beating, those of Equal temperament show unstable decaying process.

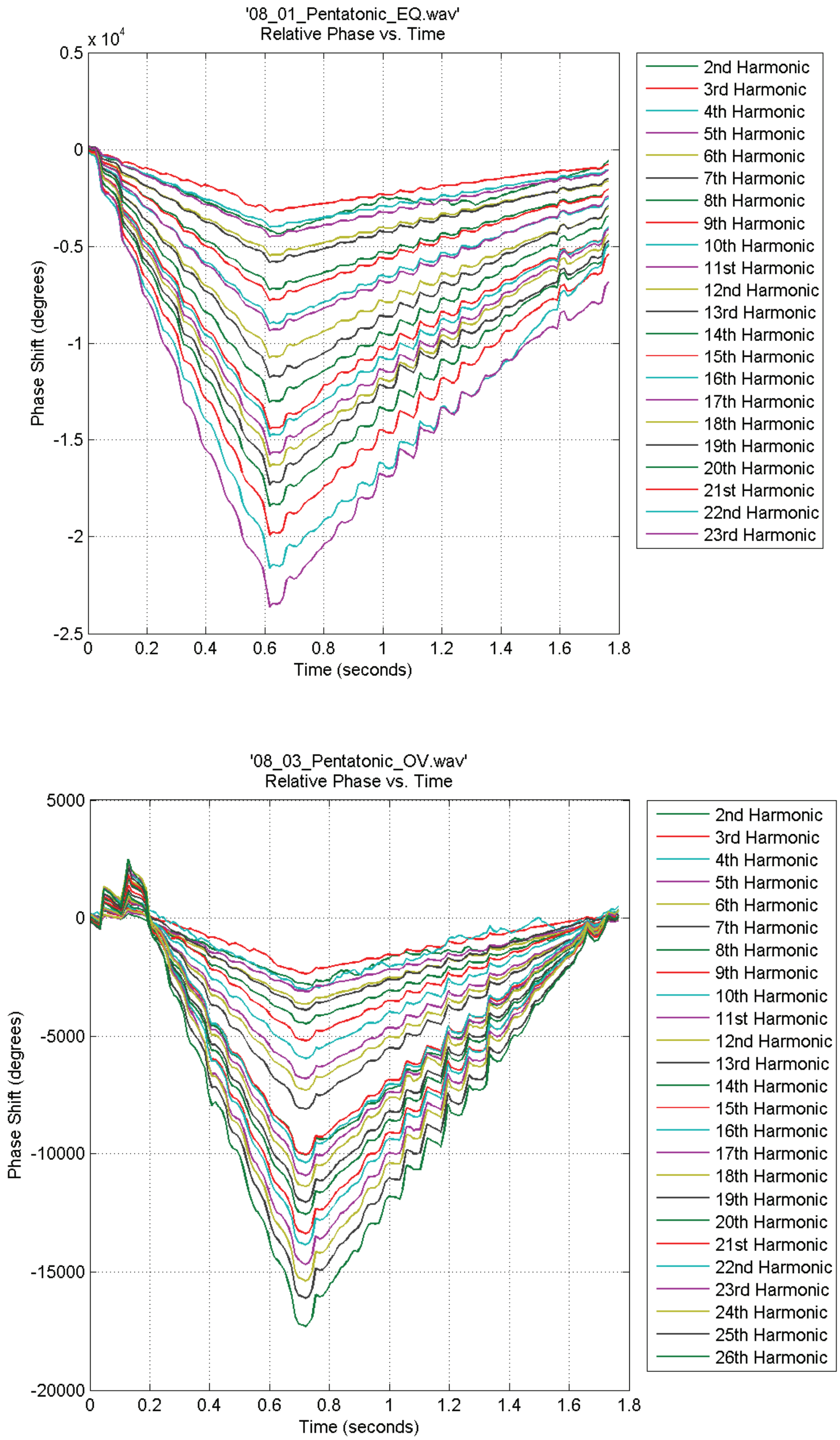


FIGURE 22 – relative phase shift degrees of pentatonic chord in EQ and JST tuning

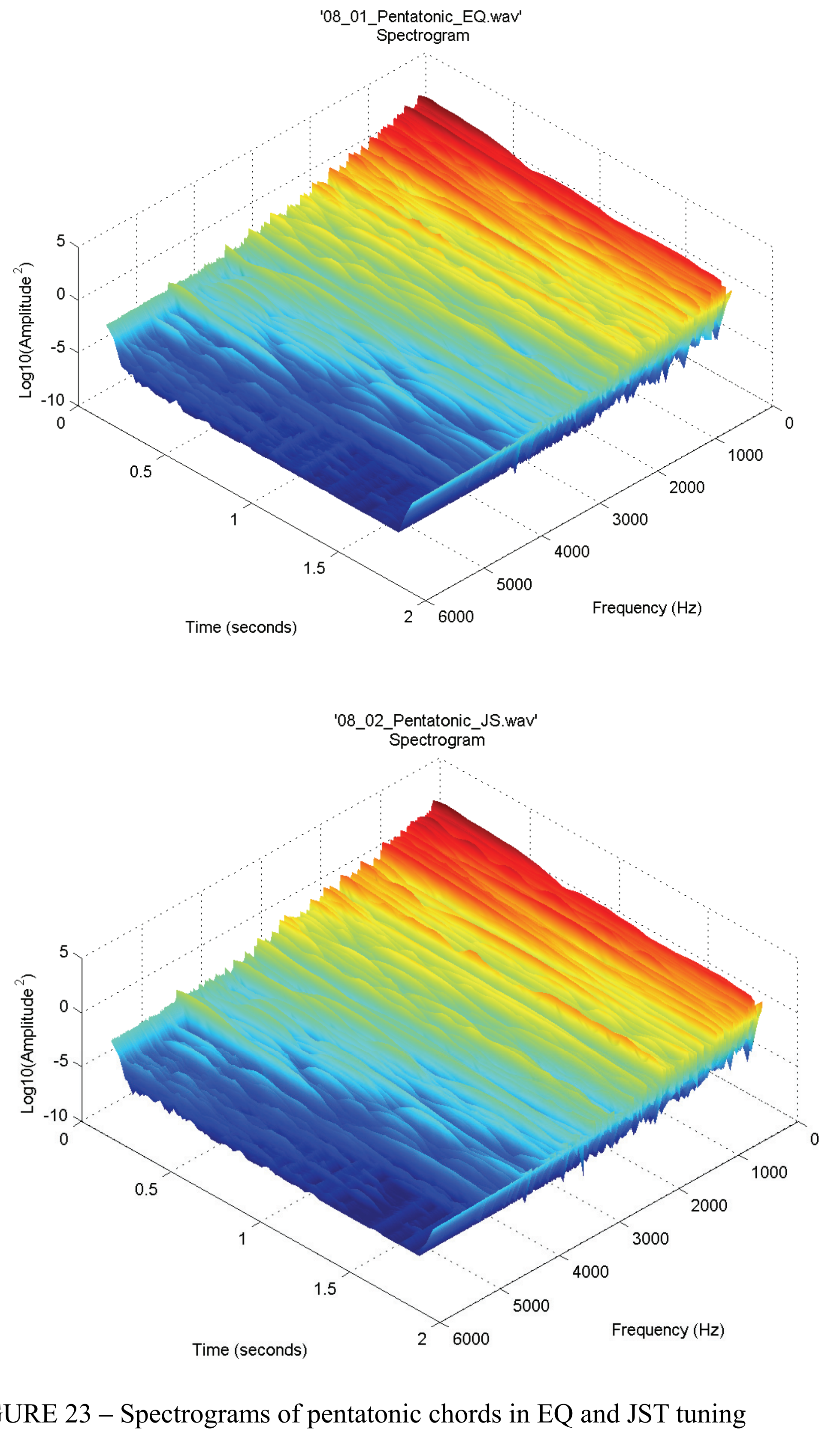


FIGURE 23 – Spectrograms of pentatonic chords in EQ and JST tuning

6. Conclusion

The expected consonances with frequency deviations between Δf and $2 * \Delta f$ of Equal tempered chord were not prevalent in the actual chords' sounds. On the contrary, in many chords Just tuned or overtone based tuned harmonies showed more stable and consonant quality than the Equal temperament based harmonies. Especially for the dominant triad, the non-conventional frequency ratio 4:7 made a striking difference between Equal temperament and overtone based tuning system, and this is also noticeable easily by ears. Even in the modern harmony-*Petrushka* chord, this overtone based tuning engendered a much more abundant and lavish harmonic spectrum, and gave a new possibility of employing multitudinous tuning ratios in the future classical compositions. In the pentatonic scale and harmony, Just tuning system showed much more lucid and crystallized quality than that of Equal tempered tuning system. This was prominent especially above the third and higher harmonics, because the deviations between fundamental ratios come out more in the frequency range. The overall results demonstrate that the quality of consonance and dissonance does not entirely depend on the limited tuning system, but on the correlations between the harmonic content, their frequency ratios, and the tuning system. For example, the consonant quality of dominant seventh chord in the overtone based tuning system was more effective than the other tuning systems, while the pentatonic chord in the Just tuning system was clearer than the other tuning systems. This witnesses the fact that the tuning system can be diversified based on the myriad of musical context, not only dominated by a single Equal tempered tuning system.

Notes

1. Steven Errede, Lecture note 8, p 10, http://courses.physics.illinois.edu/phys406/sp2014/Lecture_Notes/P406POM_Lecture_Notes/P406POM_Lect8.pdf, accessed [20140507, 00:37]

2. Henry Steward Macran, The Harmonics of Aristoxenus, Oxford at the Clarendon Press, p8

3. Thomas D. Rossing, F. Richard Moore, Paul A. Wheeler, The Science of Sound 3rd Edition, Pearson Education, Inc., Addison Wesley, San Francisco, 2002, p. 123

4. Zwicker, E., G. Flottorp, and S. S. Stevens, Critical Bandwidth in Loudness Summation, J. Acoust. Soc. Am. 29: 548.

5. Ben Johnston, edited by Bob Gilmore, Maximum Clarity, University of Illinois Press, Urbana and Chicago, p83-84

6. Georg Friedrich Haas, In Vain, Universal Edition, 2000, p163 / m.515

7. Robert Hasegawa, Eastman School of Music, Gegenstrebige Harmonik in the Music of Hans Zender, MTSNYS Annual Meeting, April 9, 2011

8. Errede, Lecture note 11, p 7-10, http://courses.physics.illinois.edu/phys406/sp2014/Lecture_Notes/P406POM_Lecture_Notes/P406POM_Lect11.pdf, accessed [20140507, 00:39]