

Acoustical Reflection: Gathering Sound from a Point Source with an Elliptical Reflector

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Abstract

The purpose of this lab was to test the validity of using an acoustical reflector in the shape of an ellipsoid to gather sound waves at a point. A sound source was placed at one of the foci of the ellipsoid and the region around the other focus was scanned with a variety of acoustical monitoring equipment. We searched for a sign of focusing at the empty focus as predicted by our theory. After many attempts, we observed modest focusing, but were unable to achieve the precise result we were expecting. We theorized the discrepancy between our results and our theory rose from the inherently sensitive nature of the geometry. Indeed, after adjusting our theory to consider our imperfect set-up, the new predictions matched the measurements much more closely. We concluded that the theory is most likely sound in nature, but difficult to achieve in practice.

Introduction

The motivation for this experiment comes from some of the notable properties of an ellipse (which can be easily extrapolated to three dimensions with an ellipsoid). First, the reflection off the surface of the ellipse from any ray originating at one focus will pass through the other focus.

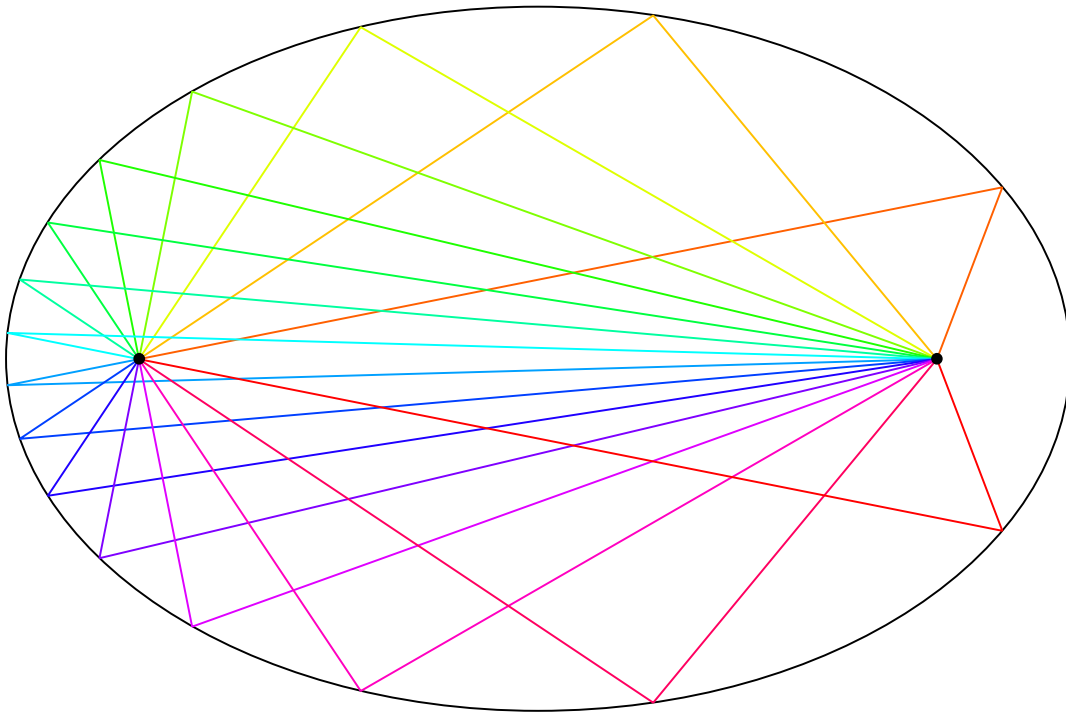


Figure 1: Demonstrating the reflective property of the ellipse.

This suggests that if a sound source were to be placed at one focus, the signal at the other focus should be very strong. Second, for any point on the ellipse, the sum of the distances to the foci is a constant. This means that not only will the signal at the empty focus be strong, but also the phase should remain intact.

These properties could be used to transmit acoustical information to a precise location using a low-power source. To make it more practical, only a small section of the ellipse (or ellipsoid) could be used, and it could still reflect most of the sound.

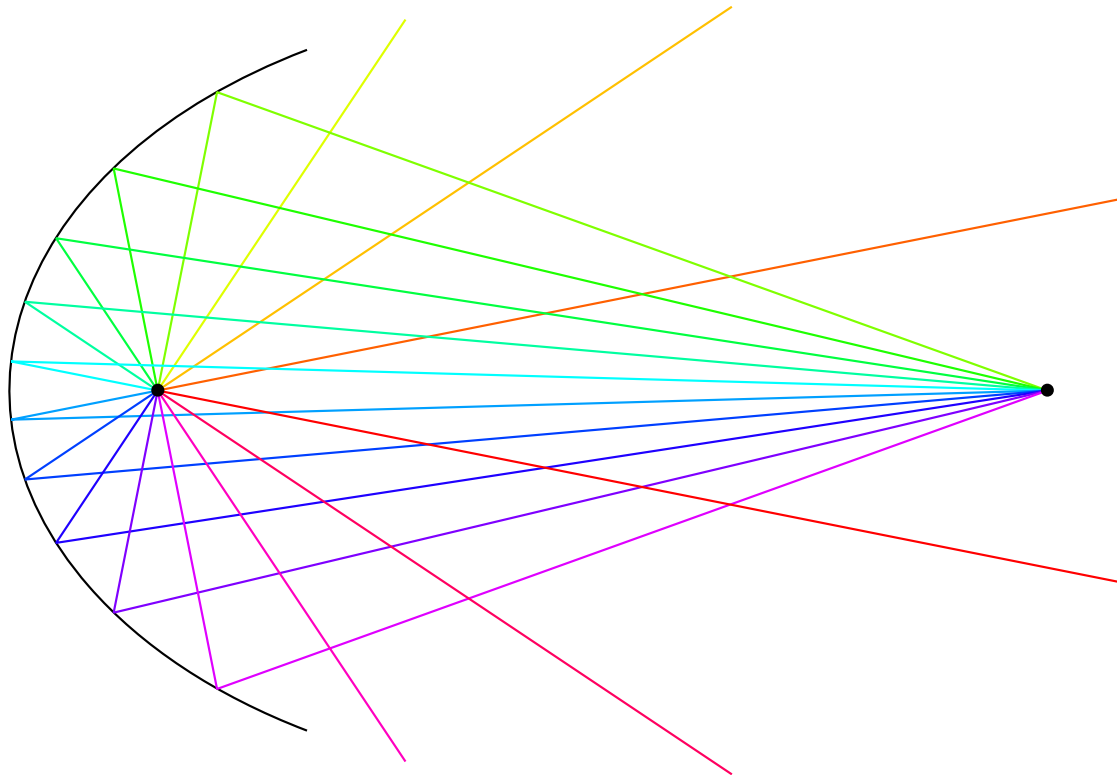


Figure 2: Using only a section of the ellipse around the source.

Alternatively, the section could be placed around the receiver so that the surface would gather sound from a precise location. Unfortunately, however, this idea went untested.

This general concept is often used with light. For example, a flashlight uses a reflector to create a beam of light from an omnidirectional bulb, and a satellite dish uses a reflector to focus

signals from a great distance to a point. There also exist some microphones fitted with a similar reflector (often used at sporting events) to pick up low-power sound from a particular direction. However, these application all use parabolic reflectors, which map parallel lines to a point and vice versa. While the nature of the parallel line projection from parabolic reflectors offers versatility in range, the more precise focusing of an elliptical reflector may be preferable in certain situations. They also have the previously-mentioned advantage of preserving phase information, unlike parabolic reflectors (not all path lengths are the same). In short, there was ample motivation for experimentation.

Theory

The proofs of the two important properties of the ellipse require extensive algebra and aren't particularly enlightening, so they won't be included. Instead, we'll look at the results of a simulation as a proof-of-concept. We'll start with a section of an elliptical reflector like in the introduction, and simulate a sound wave being emitted from the enclosed focus. Most of the sound wave will reflect off the surface and be redirected towards the other focus, while a smaller portion will escape. The parts of the wave crest that were reflected will all arrive at the empty focus at the same time, preserving phase information.

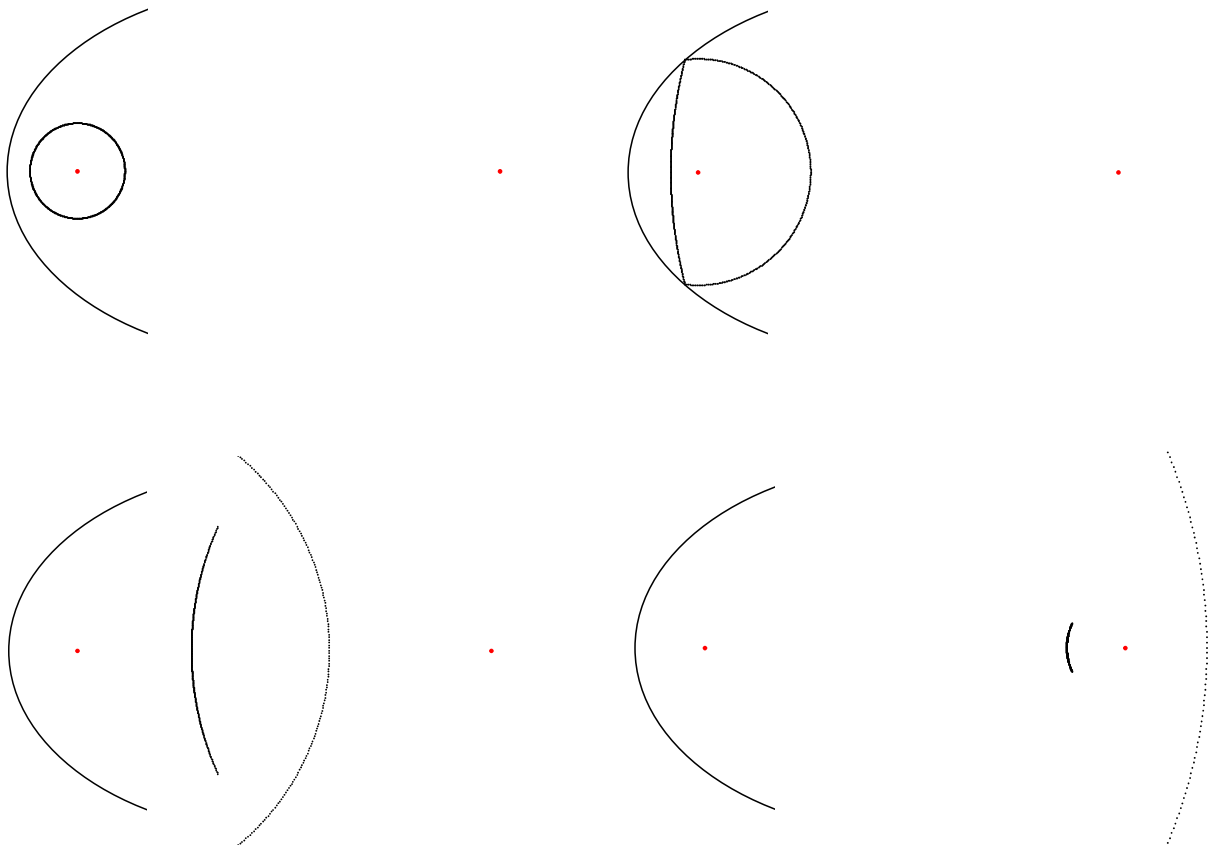


Figure 3: Various stages of a wave crest in the simulation (in the order of top left, top right, bottom left, bottom right), where the red dots are the foci.

Validating the theory in this simplified case was the first step, and its success warranted updating the simulation to three dimensions. The extrapolation is simple, as the ellipsoid in the three-dimensional model is just a surface of revolution of the ellipse in the two-dimensional model.

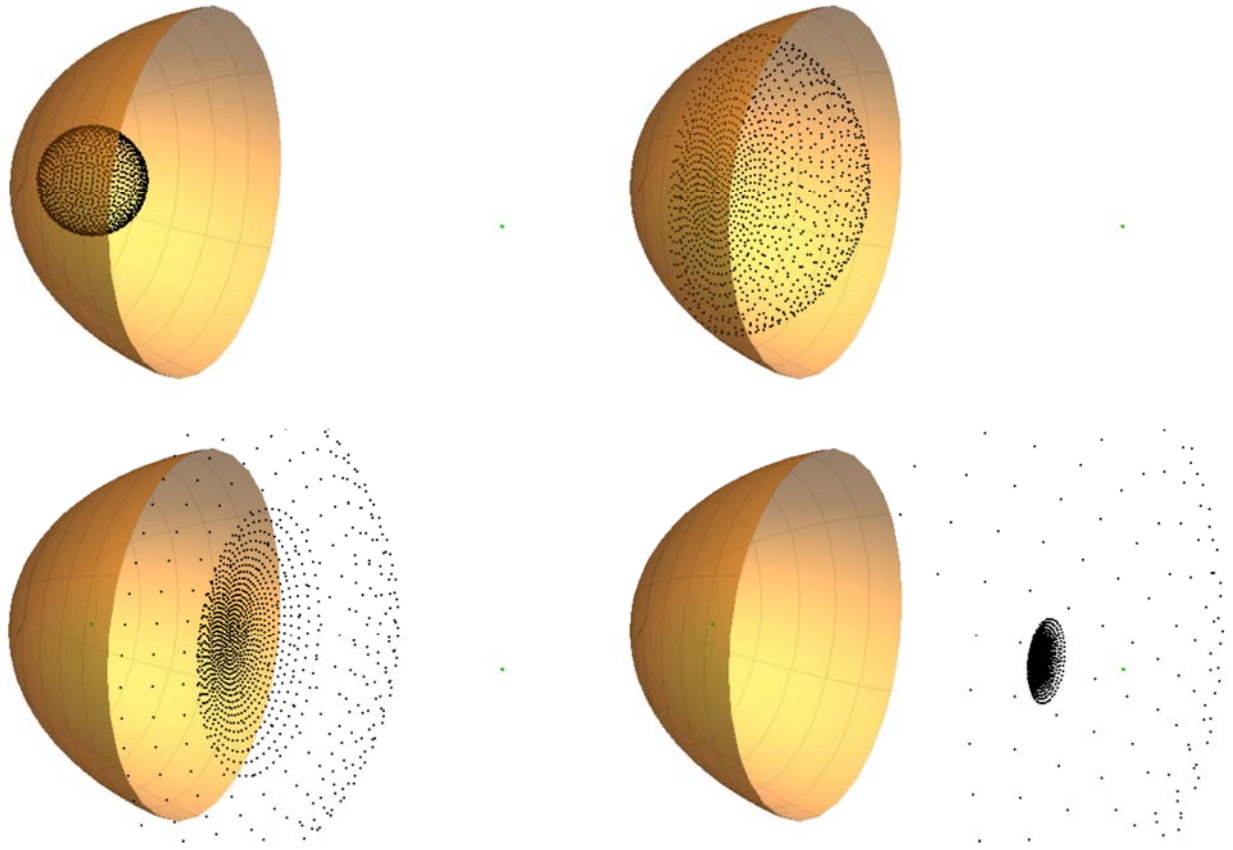


Figure 4: Three-dimensional analog of Figure 3.

Under perfect circumstances, we'd expect the theory to work perfectly as well. However, it's wise to also analyze the behavior of the reflector under imperfect circumstances, as it will be more indicative of the results of our experiment. First, we used the results of the simulation to set the dimensions of the eventual product. The eccentricity of the ellipse would be 0.75 and the semi-major axis would be 15 cm, so that the distance between the foci would be 22.5 cm. We set up the simulation with those values and allowed for an error in the placement of the sound source so that it could be slightly away from the focus. Even displacing the source by a few millimeters causes large deviations from the expected behavior.

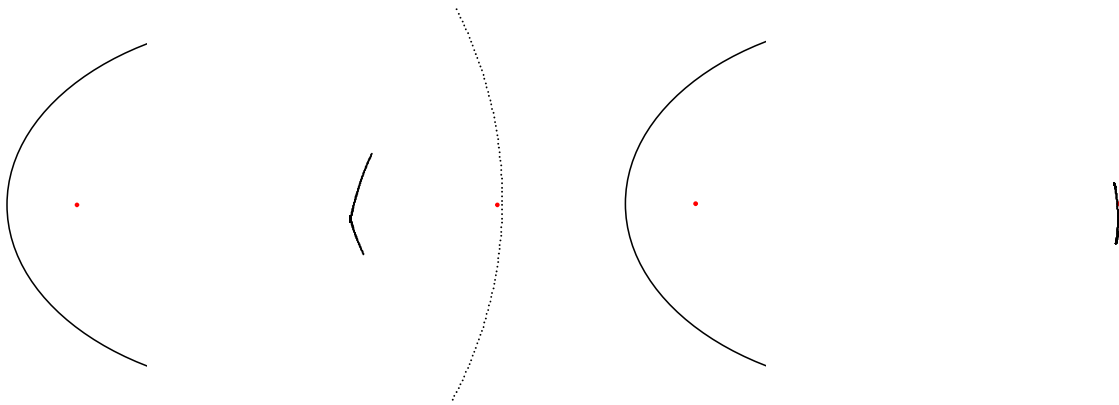


Figure 5: Result of displacing the sound source 3mm longitudinally and 2 mm transversely.

While placing the sound source exactly at the focus causes the sound wave to be reflected towards the other focus exactly, displacing the sound source by a few millimeters causes the reflected wave to be several centimeters wide by the time it reaches the other focus. Any transverse error will also cause asymmetries in the reflected wave, as seen in Figure 5 above. A slightly less severe problem is if the error is only longitudinal.

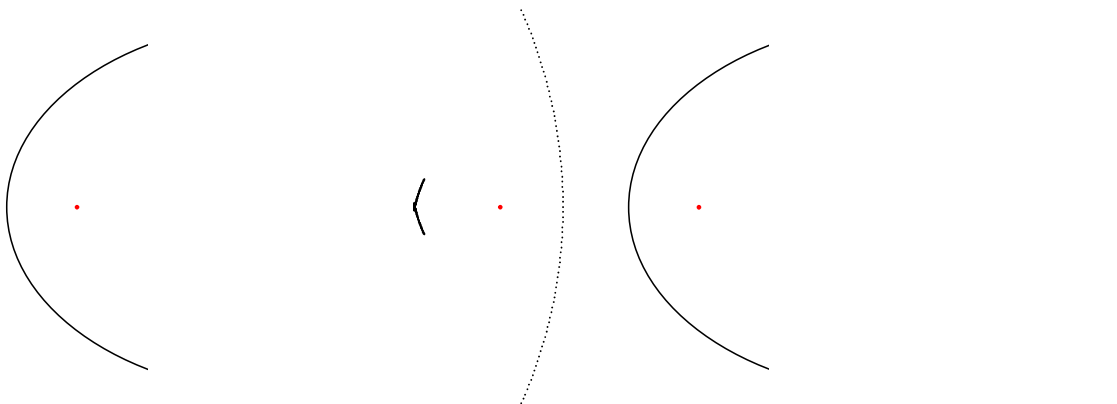


Figure 6: Result of displacing the sound source 2mm longitudinally

The reflected wave is symmetric, but it's still several centimeters wide at the other focus, rather than convergent at a point. This demonstrates that the geometry is sensitive, so that any error in the input is magnified in the output. This will be a lingering problem in our experimentation.

Experiment Procedure

The first step in preparing for the experiment was to manufacture the reflector. The model is the section of the ellipsoid from the simulation with a stand and some struts pointed towards the focus.

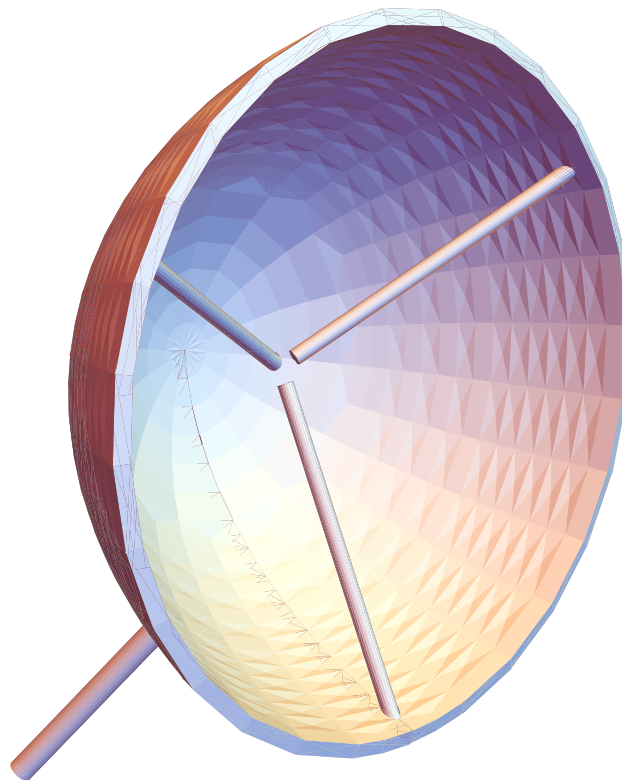


Figure 7: Stereolithographic model to be printed.

The size of the ellipsoid (22.5 cm from focus to focus) was restricted by the dimensions of the printer. As evident from Figure 7, the nature of stereolithography files approximates the surface with triangles, so the surface won't be truly curved. In addition, extensive support material is

required during printing which must be removed to obtain the final product. Smoothing the connecting regions requires patience, and likely introduces another layer of error.



Figure 8: Result of 3D printing.

Once we had the components, setting up the experiment was rather simple. A small sound source (initially an earbud) was placed at the center of the spokes, driven at a constant frequency by a lock-in amplifier. In order to avoid cancellation effects, the wavelength of the emitted sound should be smaller than the diameter of the reflector, which was about 18 cm, corresponding to a minimum frequency of about 2 kHz. We drove the earbud at 10 kHz, thinking that smaller wavelengths would produce better results than larger ones. We then

scanned planes around the empty focus with a sound pressure microphone, a particle velocity microphone, and a pair of reference microphones.

We initially scanned in the longitudinal plane (so that the axis of symmetry lied in the plane), as this would give the most information per scan due to the cylindrical symmetry. We scanned a 30 cm by 30 cm region in steps of 1 cm. Since the diameter of the reflector is only 18 cm, this region would likely be wide enough to capture the whole width of the reflected wave, and since the focus-to-focus length is 22.5 cm, it would also likely be deep enough to see the important behavior. While smaller steps would provide finer resolution, too many steps would simply require too much time to be practical.

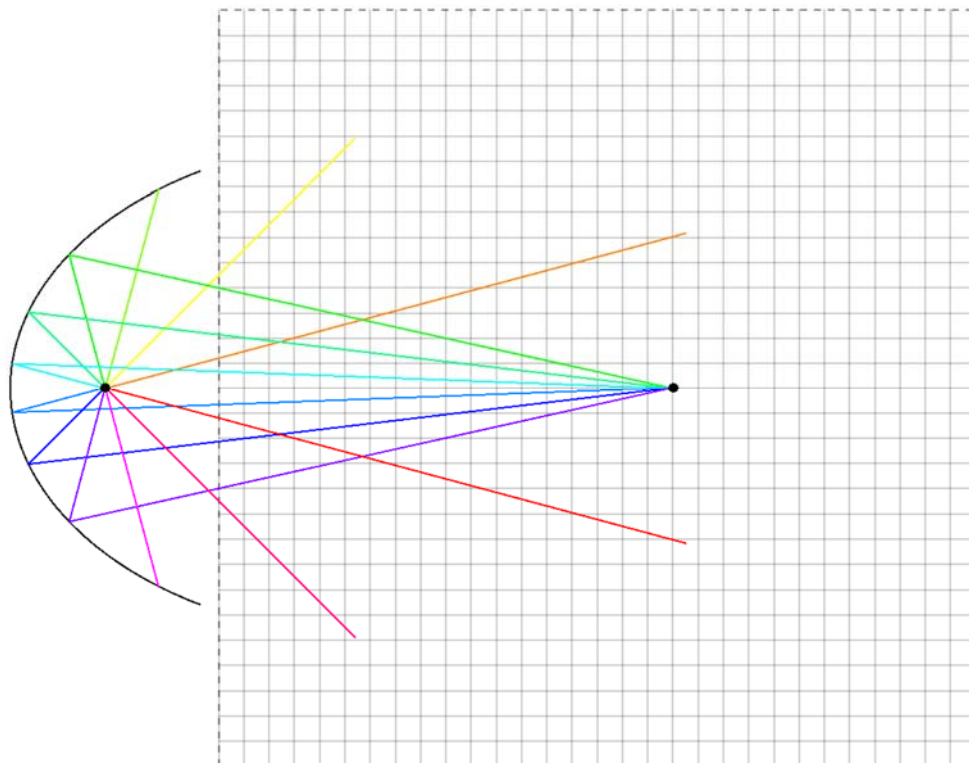


Figure 9: Scanning region for longitudinal scans, where each grid point is a point of measurement.

The rest of the experiments also used this general set-up, only substituting the source. We swapped the earbud for a piezoelectric source in hopes that it would have a smaller directivity factor. We also used a narrow tube to reduce the size of the earbud's opening to more closely approximate a point source.

Results and Discussion

We performed several scans, and each has several plots of data. The first scan we performed was a longitudinal scan with an unmodified earbud as the source. The source is in the direction of the negative x-axis. Here's the raw data:

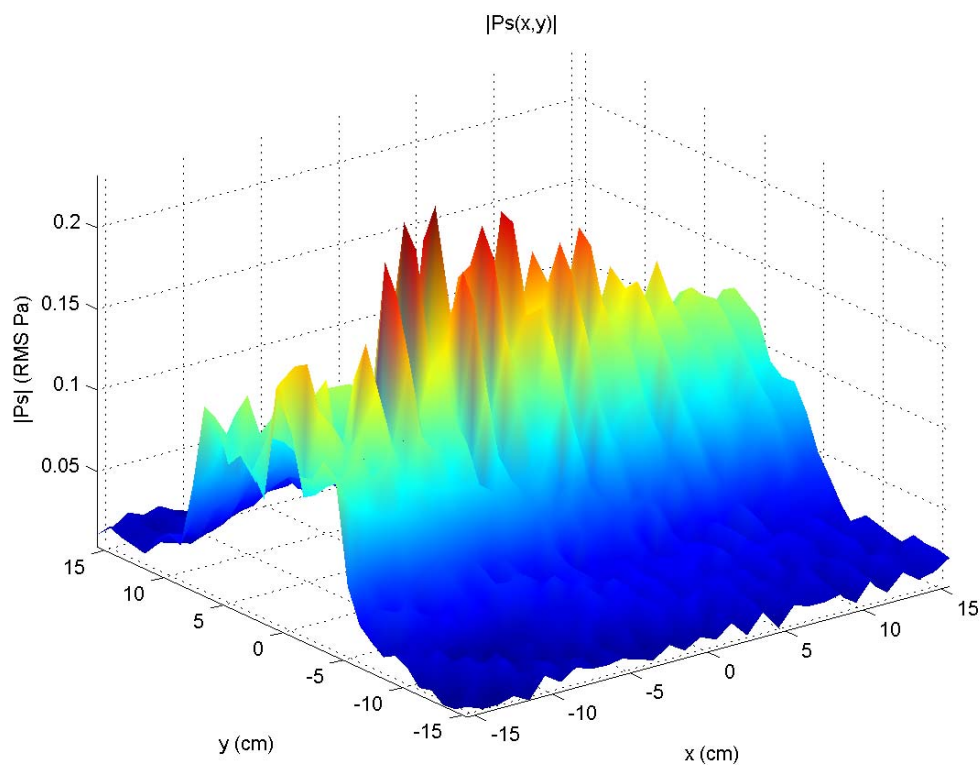


Figure 10: Magnitude of complex pressure of earbud as source.

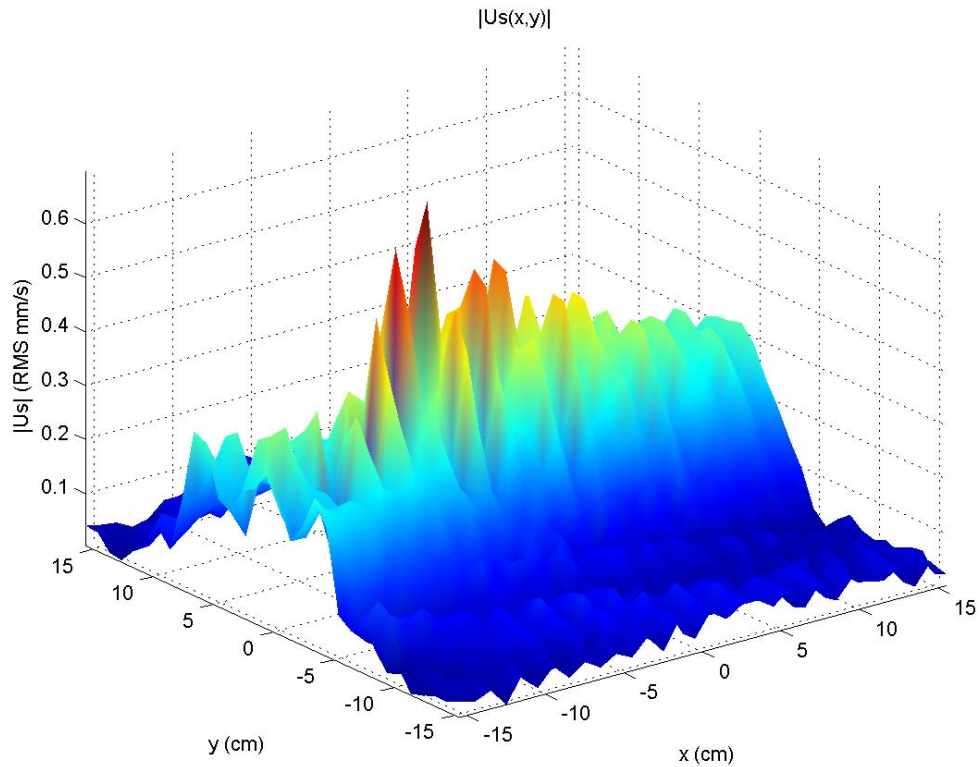


Figure 11: Magnitude of complex particle velocity of earbud as source.

The plots are unsurprisingly similar, and each has some expected qualities and some unexpected. Primarily, there is something of a peak, but it's not as dominant as the simulation suggested. The magnitude of each value gradually decreases along the x-axis beyond the empty focus, and is rather small between the source and the empty focus. Also, the low magnitude drops off considerably as the distance from the x-axis is increased. This is to be expected, since the radius of the reflector is only about 9 cm, so it's reasonable that there's not much sound for values of y beyond that. However, the simulation predicted that the width decrease to a point at the focus, but it never really gets narrower than 10 cm here.

Of course, we wanted to see clearer results, and the first thing we addressed was the sound source. Going by the clear “beam” effect that can be seen along the axis of symmetry, we theorized that the sound from the earbud was too narrowly directed, so that only a small fraction of the reflector was actually reflecting sound. We addressed this by replacing the earbud with a 1 cm diameter piezoelectric speaker. It was free of any directional housing, unlike the earbud, so it seemed reasonable to expect it to illuminate more of the reflector. We also changed the orientation of the experiment to remove a wall from the background, so now the source is in the direction of the negative y-axis.

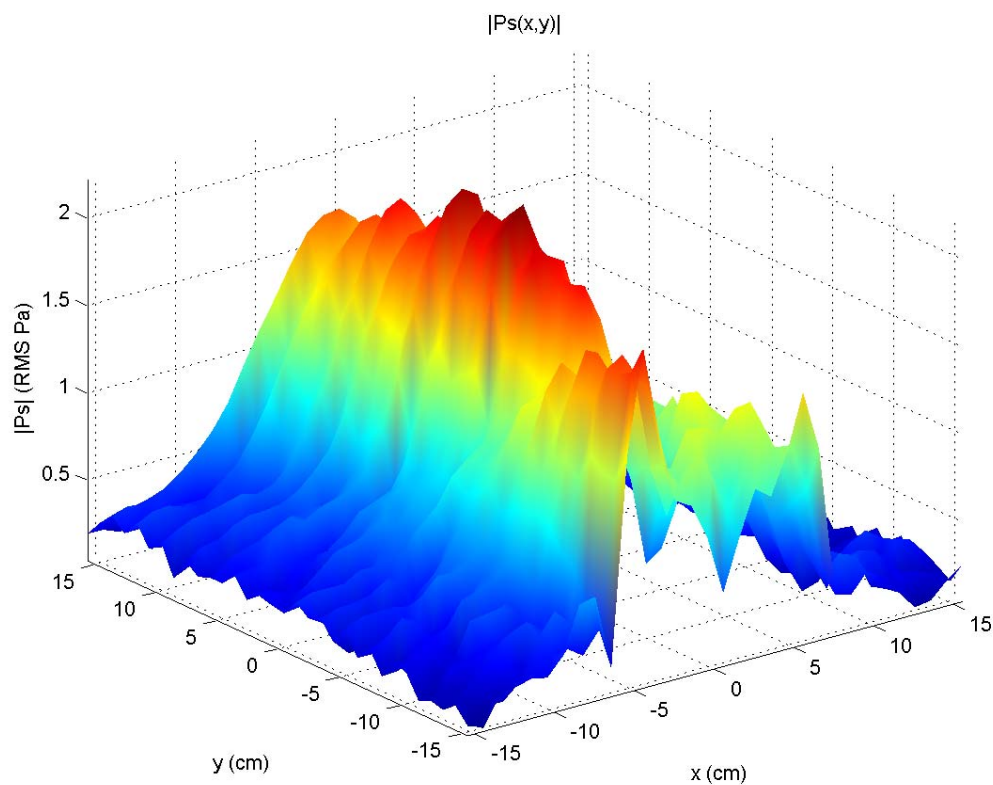


Figure 12: Magnitude of complex pressure of piezoelectric speaker as source.

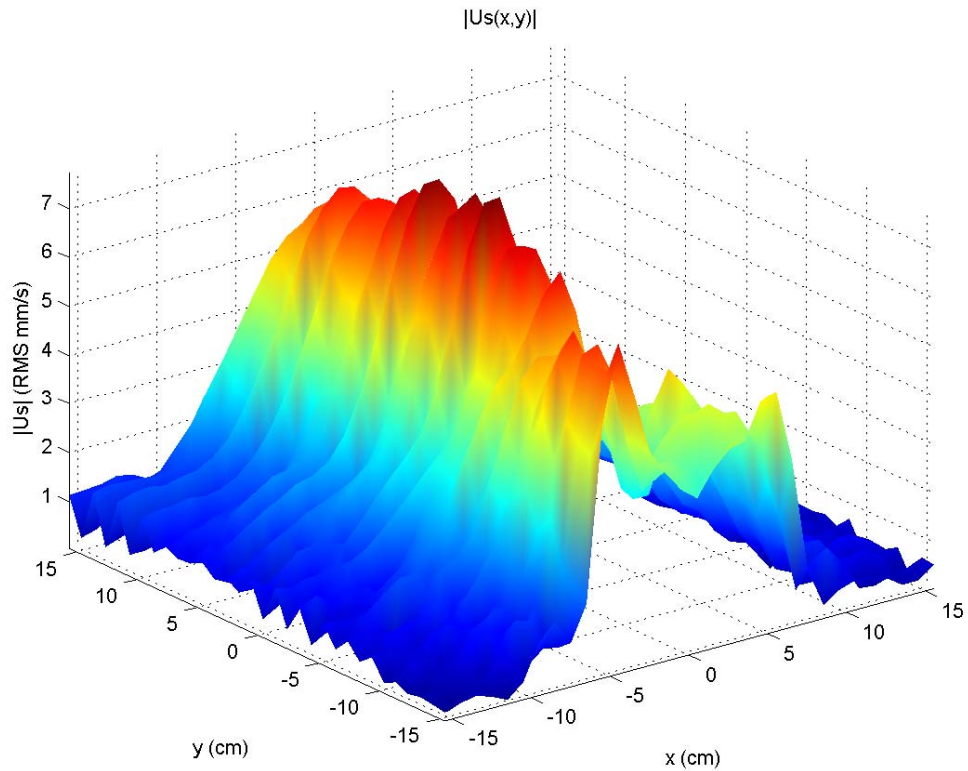


Figure 13: Magnitude of complex particle velocity of piezoelectric speaker as source.

Again, a rather indistinctive peak can be seen in approximately the correct location at $y = 3$ cm, but the result is no clearer than before. In fact, this time we can see some asymmetric behavior near the source. We were expecting the sound to be cylindrically symmetric, so this was troubling. Initially it was a mystery, but we'll revisit this with an explanation later in this section.

We decided to do a scan of the earbud by itself (no reflector) to see if directivity was actually an issue. The source is back at the negative x-axis.

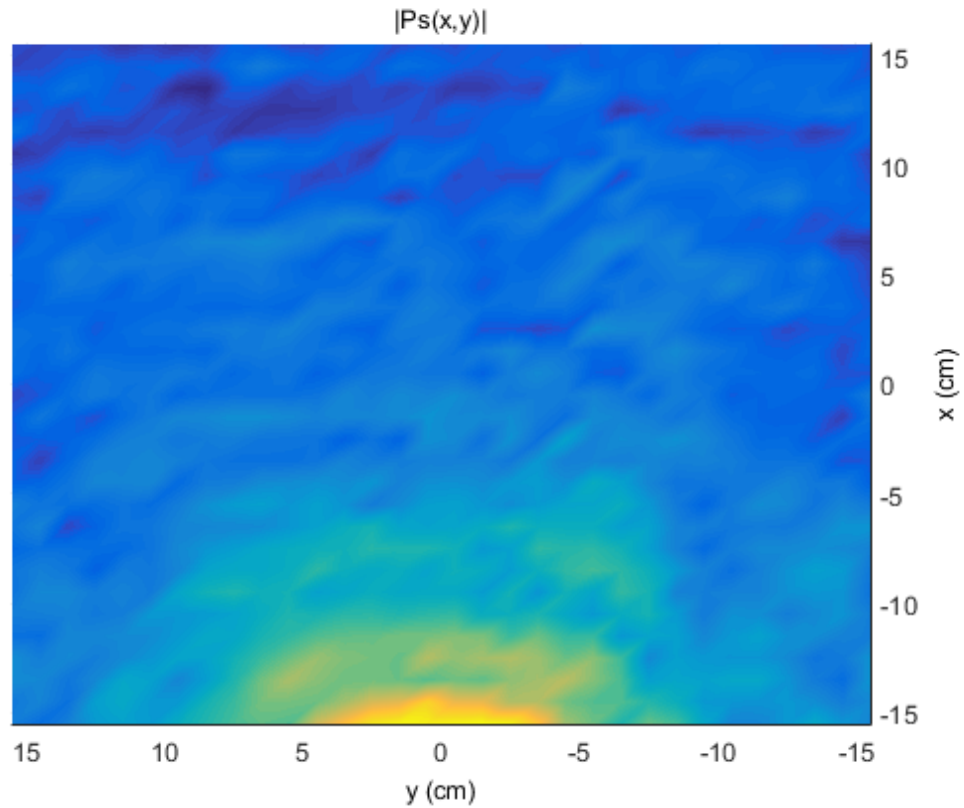


Figure 14: Magnitude of complex pressure of earbud in isolation.

In fact, the earbud seems to do a pretty good job of transmitting sound in all directions. That meant we had to find a new explanation for the fuzzy results. We postulated that the source was too large to be approximated as a point source as in the simulation. We then attached a tube with a narrow interior to the end of the earbud, essentially decreasing the size of the source.

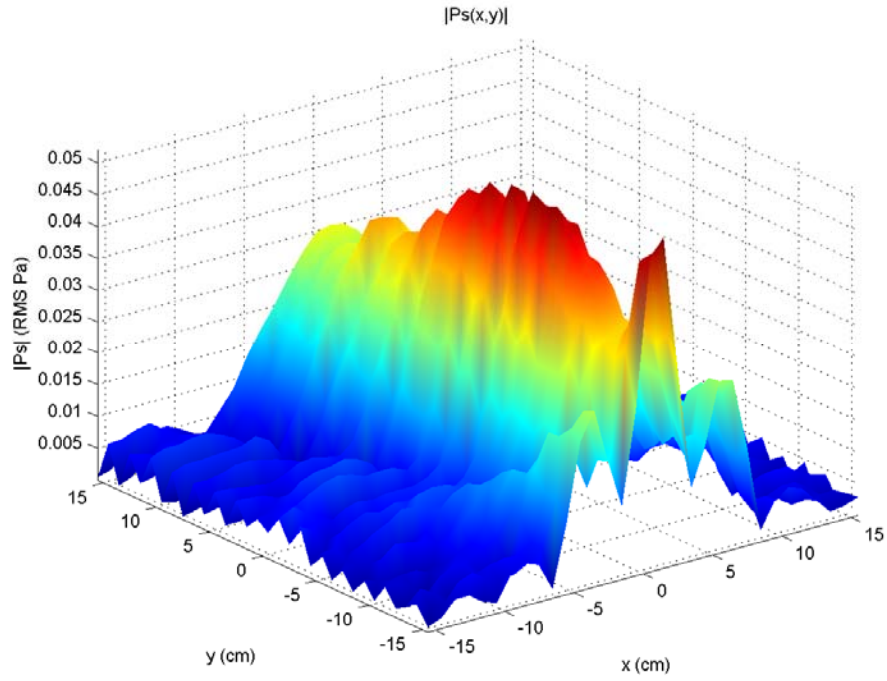


Figure 15: Magnitude of complex pressure of needle speaker as source.

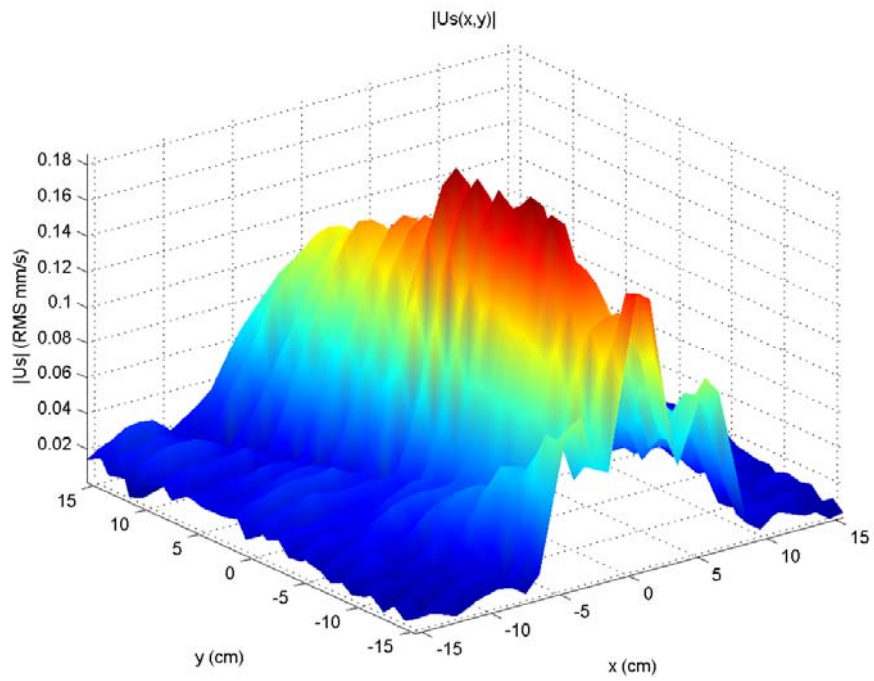


Figure 16: Magnitude of complex particle velocity of needle speaker as source.

The cylindrical symmetry appears to be back, but the peak is no more defined than it has been. There does seem to be a new feature near the source, but it's likely due to just that—it's physically near the source (adding the tube made the speaker a little longer in whole). All things considered, the results from all of the tests so far are pretty similar, so there must be something else causing the problem if the theory is to hold any water.

The most likely explanation seemed to be the sensitivity of the experiment in the first place. Since the empty focus is several factors farther from the reflector than the sound source is, any error in the placement of the source would be magnified several times in the behavior of the reflected sound. This is when we went back to the simulation and generalized it to allow for a perturbed location of the sound source, and everything seemed to make sense. The effects of a longitudinal error as seen in Figure 6 seemed to be consistent with the breadth of the peak along the axis of symmetry and the relatively wide beam seen in all scans. A transverse error as seen in Figure 5 also resembled the asymmetric anomaly in Figures 12 and 13. This seemed like a promising culprit, but, unfortunately, one that was difficult to address. Our equipment wasn't conducive to making the millimeter adjustments of the source position necessary to find the correct location, and we didn't have the time to make scans with a smaller interval necessary to get the resolution to analyze the results more accurately. We'd have no reason for increased confidence in the success of further experimentation with our capabilities, so getting the results we wanted would just be a matter of luck. Unfortunately, then, it seems like we wouldn't see the focusing we were after. However, the consistency of our data with our simulation designed to show the effects of a specific error means that we probably identified the true problem. In

that case, we still have reason to believe our theory is sound in principle, if not practically achieved with the means available to us.

Conclusion

The goal of this experiment was to test the possibility of precisely transmitting sound from one point to another. The results weren't as supportive of our theory as we would have liked, but they weren't meaningless either. We indeed saw some level of focusing at the empty focus as we hoped. We also saw that the reflected sound was cylindrically symmetric and mostly concentrated within the radius of the reflector as expected. The wide distribution of the acoustical energy along the axis of symmetry was consistent with the predicted behavior of a source slightly displaced from the focus along the axis, suggesting the sensitive placement of the source was the cause for the poor results. Unfortunately, the design of the experiment lacked the precision required to test this hypothesis thoroughly. If it is the correct explanation, then producing clear data would largely be a matter of luck, so it's not surprising that we didn't see the results we wanted. We learned that our theory has potential to work, but success would require a more precise experiment in terms of set-up and measurement.