Today:
combining SR and QM

Where we're headed:

Combining QM and GR

Cosmological implications, challenges

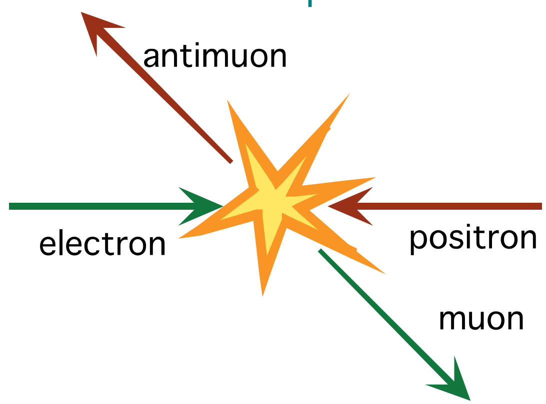
Quantum mechanics and special relativity

- There was some difficulty in combining QM and SR. Roughly, the spread in times for events required by the Uncertainty Principle leaves a tail of events (e.g. particle emission-absorption) seemingly backward in time. In 1928-1930, Dirac found a replacement for the Schroedinger equation, consistent with SR.
  - It required that for every particle that carried some conserved quantities, there be another possible particle with the same mass but opposite values of those “charges.” (“antimatter”)
    - Instead of saying a particle went backward in time, you get the same physical results with its antiparticle going forward in time.
  - Dirac was so surprised by this that at first he didn’t believe it and thought the theory was somehow relating the proton and electron, although with different masses that couldn’t have worked..
  - The positron (the electron’s antimatter partner) was found in cosmic rays by C. Anderson in 1932.

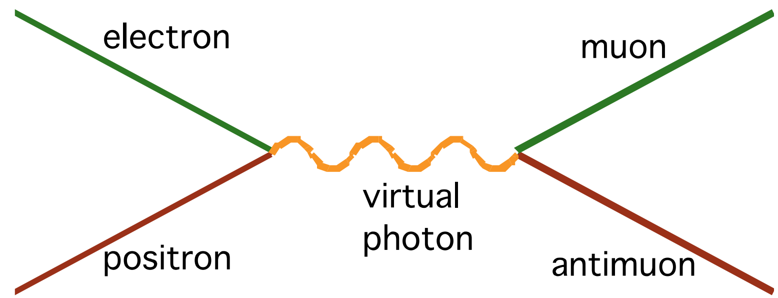
Matter and antimatter can annihilate

- each other, and new forms of matter can be produced, as illustrated by "Feynman diagrams". For example, electrons and positrons can annihilate and produce muons and antimuons:

Annihilation process:



Feynman diagram:



- *Any* particle-antiparticle pair (nothing special about muons) can be made as long as there is enough energy for $E = mc^2$.
- How do we choose, in the example above, to say if the electron-positron or the muon pair are the "basic" particles?

Building Blocks of matter?

- Should we still think of elementary particles as “building blocks” of matter?
- *They are not permanent constituents.*

- They are more like packets of conserved quantities. Those can be repacked in different ways.
- We can think of the “particles” as excitations of the fields of the vacuum.
- That’s one way to understand why electrons are truly identical- they aren’t so much “things” as modes of behavior of the vacuum.
  - E.g. if a simple harmonic oscillator (mass on spring, SHO) has one quantum of energy (hf) above the ground state, we don't ask "which quantum of energy?" If it has 10 quanta, we don't ask "which one is first?" because the quanta just describe states of the oscillator, not some preexisting "things". So our story has shades of the ether- but not an ether which prefers one reference frame!

Carrying the SHO analogy farther

- An SHO can't just sit in the position of lowest potential energy. The Uncertainty Principle forces it to spread out some, so there's some potential and kinetic energy (hf/2) in the ground state. If you "look" with a beam of high-energy particles (i.e. "measure" the position) you will not usually find it to be at the position of lowest potential energy.

- Likewise there is necessary uncertainty in the fields which specify how many particles/antiparticles of any type are present, even in the lowest energy state, i.e. the vacuum.

- The lowest energy state is not one in which we can say with certainty that nothing is there. There is a nonzero probability of finding stuff there if you look. The reason is that the quantities that describe electron-positron pairs are not all simultaneously measurable.
(i.e. they're like position and momentum of SHO, or x and y spins of an electron)

- If there were definitely zero particles, then the values of all these quantities would be definite, in violation of the uncertainty principle.

Toy Pictures of the Vacuum

- It is tempting to think of the vacuum as being filled with particle-antiparticle pairs, blinking in and out of existence, vacuum “bubbles”:
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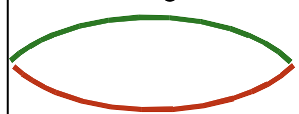
> This isn’t too bad an image, but be careful that what one actually measures depends on what experiment is done.
> *Nothing is really changing in time in this vacuum*.



- How can a state with nonzero probability of the existence of an electron-positron pair (with $E=2m_0c^2$) possibly be lower energy than a state without it?
- The uncertainty principle has a time component that is similar to the space component:

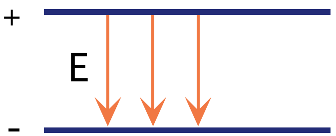
$$\Delta p\,\Delta x \geq \hbar/2 \qquad \text{Space component}$$

$$\Delta E\,\Delta t \geq \hbar/2 \qquad \text{Time component}$$

$$\Delta\left(mc^2\right)\Delta t \geq \hbar/2 \qquad \text{Lorentz invariant form}$$

- So on sufficiently short time scales, the mass (i.e. number of particle-antiparticle pairs) must become ill-defined.
We call this uncertainty in the numbers “the presence of virtual particles”.
The fluctuations in more massive particle fields (bigger m's) require smaller Δt's.
We say, loosely, that virtual massive particles can only exist for shorter times and are thus more rare than less massive ones. The “more rare” part is true.

Does this view of the vacuum makes sense?

- **Sparking of the vacuum**.
  

  - Consider a region of extremely strong electric field. If there is any residual gas in the gap, one is likely to get a spark. Suppose we have a perfect vacuum. The vacuum isn't empty, so if there is enough energy stored in the field, that energy can be transferred to the virtual pairs, making them real. The required electric field is enormous, and this effect has only been seen in the electric field near super-heavy nuclei (Z ~ 180)
- **The Casimir force**.
  - Take two plates of metal near each other. They do not allow any long-wavelength E-M excitations in between. The lost excitations have positive energy. As the plates get closer, they eliminate more positive vacuum energy. Hence closer means lower net energy- so any two good conductors *attract* when they are nearby. (repeatedly measured, to fairly good precision)
- **Hawking radiation**. (no experiments yet!)
  - Near black holes, the gravitational field is very strong, and the energy stored in the field can be converted into particle pairs in a similar way. By this mechanism, black holes are predicted to "evaporate." (The evaporation time for a solar mass BH is longer than the age of the universe.)

So empty space isn't.

- Combining SR and QM requires that the number of particles of some type be represented by a quantum field. It is therefore subject to uncertainty relations. We treat the vacuum as if it were filled with a sea of potentially existing particle-antiparticle pairs, giving a "zero-point energy", like the (measured) zero-point energies of SHO's.
- This treatment is not just hypothetical. The vacuum energy in the E-M field depends on confinement by conductors. Therefore a force is exerted on the conductors (the Casimir force). It's measurable, rather precisely.
- A force depends on differences in energy as a function of position. So measuring the Casimir force (e.g.) does not tell you the energy density of empty space, only how it changes when pieces of metal, etc., are moved around in it.
- Absolute energies (as opposed to differences) enter into physics as the masses which give rise to gravity. So we have to ask whether we can calculate the energy density in space that serves as a source of gravitational effects simply by adding up the sorts of background energies that give the Casimir effect.
- <u>If we do so, we end up with an infinite energy density</u>, just like in the old black-body radiation problem. That should be rather noticeable- e.g. in its effect on cosmology!
- So obviously <u>we're missing something</u> in our understanding of how to treat this problem.

A problem in Cosmology

- Individual masses are pulled together by gravity. A uniform collection of masses would make space itself pull together, i.e. decelerate the expansion. What would happen if there were some (finite) mass-energy density associated with the mere existence of space, i.e. a fixed density rather than a fixed mass?

Here's a crude argument:

- Suppose you have a Newtonian gravitational problem, with fixed mass M spread out over dimensions R. The gravitational potential energy will be something like (not worrying about factors of 2,…)
  - $-GM^2/R$
- The forces are the derivatives of energies wrt position, so these are of order
  - $-GM^2/R^2$
- Producing inward accelerations of order:
  - GM/R^2.

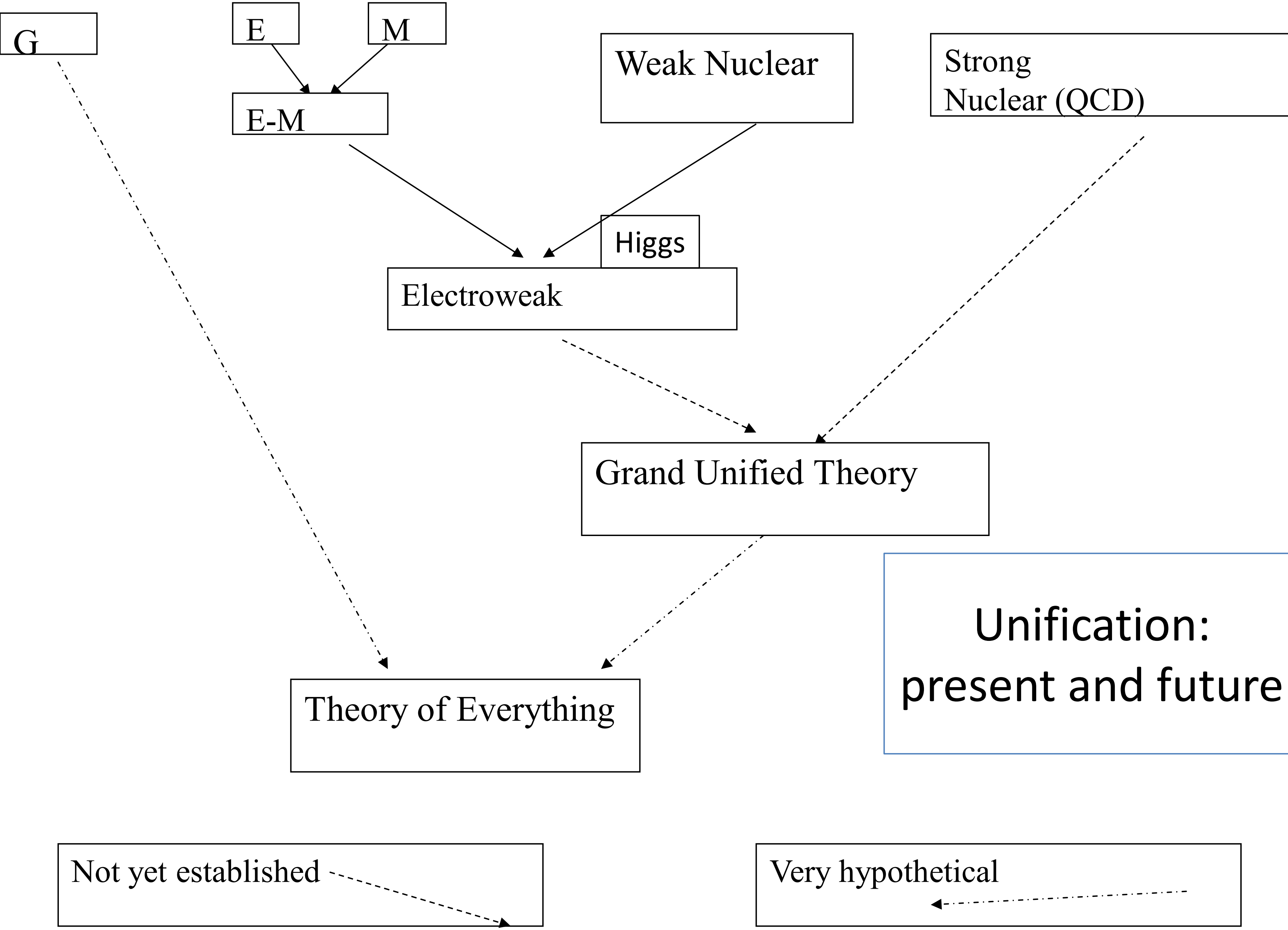
- Whether that is an important inward acceleration depends on the mass density, which is decreasing as things expand. There's a critical density (for a given expansion rate) below which the outward expansion never stops.

Gravity acting on Fixed-Density

- Now what happens if the thing that's fixed is the density
  - $\rho=M/R^3$?
- The potential energy is of order
  - $-G\rho^2 R^6/R = -G\rho^2 R^5$
- So the force (given by the derivative wrt R) is of order
  - $G\rho^2 R^4$ *outward*.
- Dividing the *outward* force $Gr^2 R^4$ by the mass $r\,R^3$ gives outward acceleration:
  - $G\rho R$.
- The second derivative of R wrt time is proportional to +R.
- R will grow (or shrink, or some combination) exponentially!
- It turns out that this odd Newtonian fixed-density calculation captures the key behavior of the GR equations, which also show *exponential growth of space* if the density rather than the mass is fixed.
- This effect is known as inflation. It is speculated that intense inflation occurred about 10^{-33} sec after the big bang- for reasons we shall soon discuss.
- We shall see strong evidence that the universe is weakly inflating now!

Unification of forces

- There are (conventionally) four known forces:
  - gravity
  - strong nuclear (QCD)
  - electromagnetism (QED)
  - weak nuclear.
- Except gravity, *all* the forces seen in elementary physics problems (" sliding friction, static friction, normal, spring, rope, etc.) are manifestations of QED. QCD binds the nucleons together; the weak nuclear force produces some radioactive decay.
- You've seen one unification already: electricity and magnetism. These seem like entirely separate phenomena at a *long* first glance (say from 500 BC to 1700 AD)
- Links between them were found:
  - Moving electric charges exerted forces on magnets.
  - Moving magnets exerted forces on electrical charges.
- The *unified* theory (Maxwell's equations) does more.
- It predicts the existence of a new type of wave: E-M waves. The properties of that wave (speed, polarization, energy density, momentum) all are *derived* from the theory unifying the forces. In this case, the predicted new wave wasn't so new- it was just light. In other cases, the predicted new waves/particles from unifications are usually new enough that one must do new experiments to find them.
- <u>The unified theory is more than the sum of the parts.</u>


G
E
M
E-M
Weak Nuclear
Strong
Nuclear (QCD)
Higgs
Electroweak
Grand Unified Theory
Theory of Everything
Unification:
present and future
Not yet established
Very hypothetical

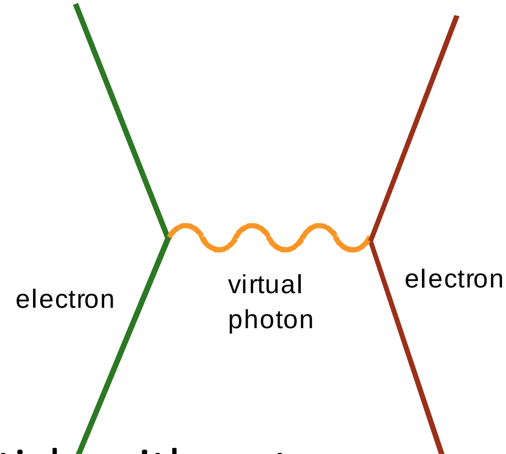
Dis-Unification: the E-M example

- In atoms, electrical forces are much stronger than magnetic forces. At temperature so high that relative velocities on the order of c are common magnetic and electrical effects are of about the same size. In a world of charged particles flying around near speed c, it would not make sense to have separate treatments of "magnetic" and "electric" forces, since these would obviously be manifestations of the same effects. Remember that these fields transform into each other in Lorentz transformations.
- Magnetic and electric forces become separable only if you pick a particular reference frame in which most things aren't moving fast. I.e. if the local environment strongly breaks the Lorentz symmetry of relativity, the forces look different. *In the full symmetry environment, they obviously belong to a unified theory.*

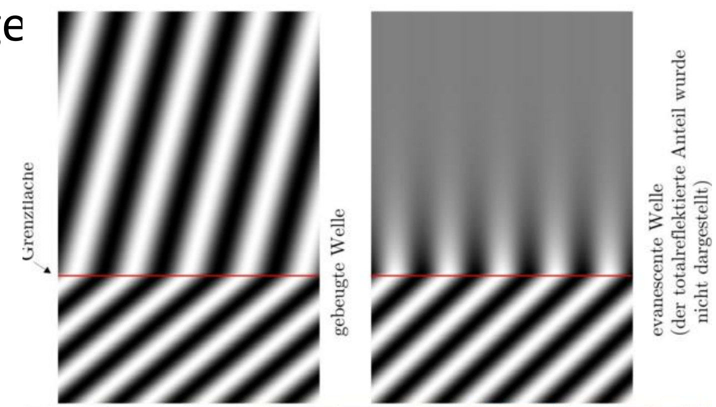
- These features- prediction of new particles, connection between symmetry loss and apparent disunity of forces- will be found in the other unifications, including the one fully achieved (electro-weak), the one with some serious proposals and experiments (grand unification), and perhaps the one in embryo ("theory of everything").

Forces and Particles

- One can regard the EM force to be the result of the exchange of photons. For example, one can draw this picture to represent one electron bouncing off another one :
  - Conservation of energy and momentum does not allow the photon to be “real” – no E=pc conserves both E and p here.



- Let’s look at a force involving exchange of a particle with rest mass m.
  - $p^2c^2=E^2-m^2c^4$.
  - For long times, $E^2=0$, so $p^2=-m^2c^2$. $p=imc$. That stands for an *exponentially decaying function* with range ~ $h/|p|=h/mc$. Forces corresponding to exchange of massive particles are short-range
- Photons have no rest-mass, so the solution for the photon field is longer-ranged, with force falling off as $1/r^2$. The “evanescent” light wave in glass on the right behaves like one of the decaying fields.



Forces and Particles

- There is no longer the old distinction between the fields that carry force and the particles that are acted on by these forces. They are all quantized fields, whose quanta are given particle-like names.
  - This idea is the completion of the thought that began with the discovery of the particle-like properties of E-M radiation and of the wave-like properties of particles.
- The remaining distinction is between <u>bosons</u> and <u>fermions</u>.
  - Bosons (integral spin) can pile up in single-particle states, and make nearly classical fields.
  - Fermions (half-integral spin) are limited to one per state, and thus are more like classical "matter", of which people used to say "no two things can be in the same place at the same time."
  - This spin-statistics connection follows from a theorem based on SR+ QM, not just an observation.
- Although we've thought of bosons and fermions as entirely distinct, getting that vacuum-fluctuation energy to not blow up to infinity may require "supersymmetry"- boson/fermion pairs whose high-energy fluctuations cancel.
  - Not yet found.

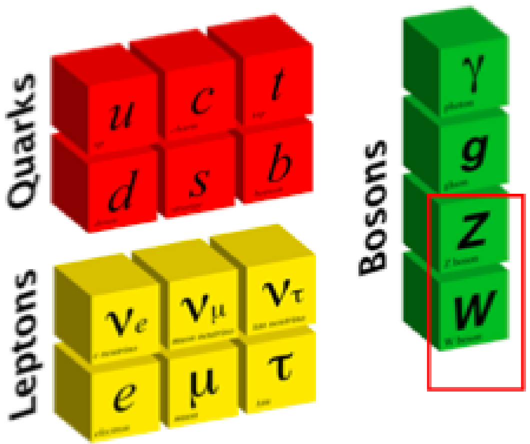
More unification

<u>The electroweak interaction</u>: EM and weak don't have much obviously in common:

- EM interactions depend on the electric charge, a conserved quantity. Weak interactions depend on another property (called weak isospin) that isn't conserved.
- EM interactions are long range ($1/r^2$). Weak interactions are short range ($\sim 10^{-18}$ m, decreasing exponentially).
- Some particles (*e.g.*, quarks) participate in EM, but not weak interactions, and some (*e.g.*, neutrinos) vice-versa.
- But there are similarities. Within its short range, the weak interaction is actually the *same strength* as EM. It has been known since 1935 (Yukawa) that the short-range forces are due to the exchange of *massive* particles. Therefore, in the late 1950s it was proposed that the weak interactions are mediated by heavy intermediate vector bosons, related to photons**. These have been found.**
- **But there's a problem: need Higgs!**

Why Need Higgs?
(slides from LianTao Wang, U. Chicago)

The Standard Model before 2012



$m_{\mathrm{W,Z}} \simeq 10^2\ \mathrm{GeV}$

- Electroweak symmetry breaking (EWSB).
  - Weak interaction has finite range

$$V_{\mathrm{weak}}(r) \approx \frac{e^{-r/r_{\mathrm{W}}}}{r},\ r_{\mathrm{W}} \approx m_{\mathrm{W,Z}}^{-1} \approx 10^{-17}\ \mathrm{m}$$

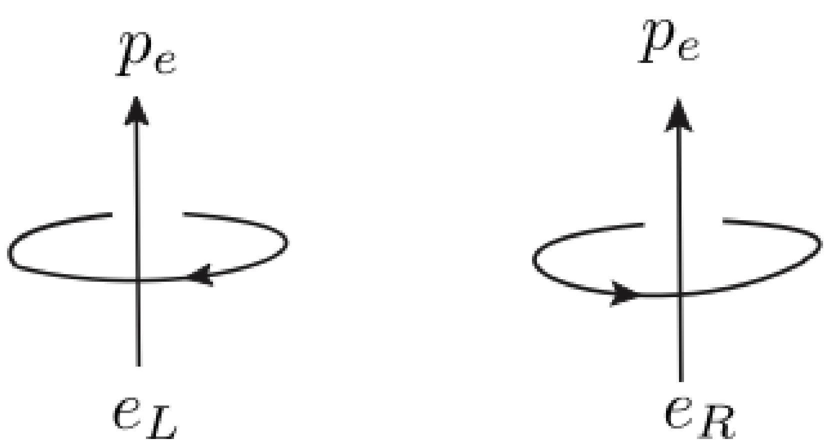
Fermi, 1934

Why Need Higgs?
(slides from LianTao Wang, U. Chicago)

Weak interaction and parity

helicity:

Left (right) handed



- Only left handed electron, e_L, has weak interaction. (fixed by symmetry)
- Parity violation. Lee and Yang, 1956

Why Need Higgs?
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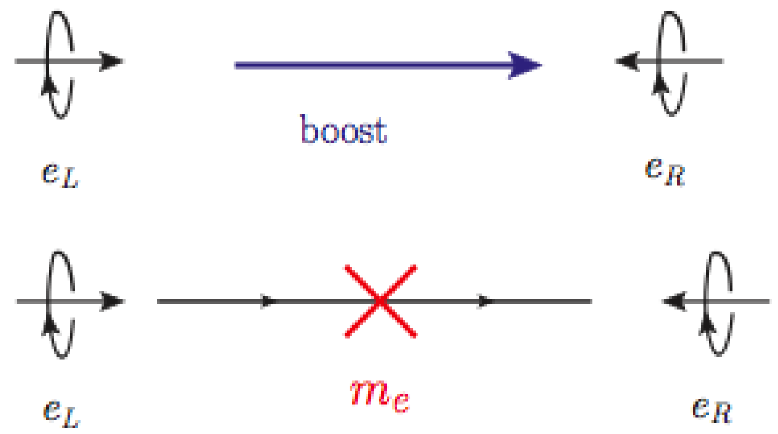
EWSB and origin of mass

Lorentz invariance:



Massive electron must have both e_L and e_R

with weak interaction

with no weak interaction

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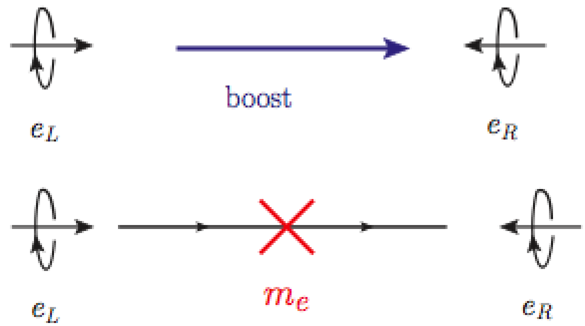
EWSB and origin of mass

Lorentz invariance:



Massive electron must have both e_L and e_R

with weak interaction

with no weak interaction

- Whatever generates m_e must break the symmetry of weak interaction.

- As a result, it will give $W^{\pm}$, Z masses as well

GUT (grand unified theory)

- The next unification would (probably) be of the electroweak with the strong nuclear force, which is described by QCD (quantum chromodynamics). There are a variety of different proposals for GUT. Each involves distinct experimental predictions for new particles, how the strengths of the interactions depends on the length scale, etc. High-energy accelerator physics is largely concerned with finding the proper form of the GUT by sorting out these effects.

What about quantum gravity, i.e. QM +GR?

- The first of the fundamental forces to be found is the hardest to integrate into the unified framework of forces in QM. We know for sure that QM and GR don't combine in an obvious way- trying to simply quantize GR gives zero or infinity for any calculation
- Thus finding some deeper theory than GR is *not* just an optional whim on the part of people who like unified theories. The present form of GR and QM are *not consistent*, so there must be some deeper form which applies in the realm where quantum effects become important (very short times/distances, high energies).
- So far the main proposals that look like they have a chance to give GR in the usual regime without making contradictions in the high-energy regime are proposals involving more space-time dimensions: string theory and its relatives, theories of interactions in 10 or 11 space-time dimensions. This approach has not yet given a complete consistent theory- i.e. one where you can write down some complete axioms, and then calculate some predictions. (Other approaches include “loop quantum gravity.)

- But there are some very suggestive and strange possibilities.

QM + GR: not just inconsistent

- Reminder: When Einstein argued that a careful experiment could violate an uncertainty relation (and hence violate QM) Bohr showed that inclusion of GR effects preserved the uncertainty, and saved QM.

- But now we've heard that to preserve the QM framework, GR must be replaced on a small scale, and as an ultimate theory. What then will rescue the uncertainty relations on that small scale?

- Will it be necessary to replace *both* QM and GR? Will that help with the measurement question?