Wave-Particle Duality & the Two-Slit Experiment: Analysis

In the last lecture, we saw that in a two-slit experiment electrons seem to behave partially like particles (in that they arrive at the scintillating screen as discrete objects) and partially like waves (in that they seem to show interference effects). The quantitative relation between the wave and particle aspects is given by the de Broglie relation

$$\lambda = \frac{h}{p}$$

(and also the Einstein relation $E = h\nu$). How can the same "object" be thought of simultaneously as a wave and as a particle?

According to the standard Born interpretation of the wave function, $\psi(x)$, its square gives the probability per unit volume of finding the electron at point x. Suppose now that we want to describe in this way a particle known, with fair confidence, to be located somewhere between

$$x_0 + \frac{\Delta x}{2} \quad \text{and} \quad x_0 - \frac{\Delta x}{2},$$

i.e., in a (1D) spatial region Δx.* Then we must ensure that $\psi(x)$ vanishes, or at least is quite small, outside this region. But clearly a plane wave, with a single wavelength λ, does not have this property; it extends over all space! So it is necessary to make up a *wave packet*, that is, a linear superposition of waves with nearly but not quite the same wavelength, so that destructive interference occurs in the unwanted regions. That we are allowed to consider such a "packet" is guaranteed by the superposition principle; however, in general, it will be dispersive, i.e., spread in space as time goes on. If, however, we look at it at some particular time and define $k = \frac{2\pi}{\lambda}$, then it turns out that the minimum spread in k (or $\frac{1}{\lambda}$) we need to produce a packet localized with spatial extension Δx is of order $\frac{1}{\Delta x}$. (This is a rigorous mathematical result if precisely formulated.) But we know that

$$p = \frac{h}{\lambda} = \left(\frac{h}{2\pi}\right) k \ (\equiv \hbar k)$$

and hence

$$\Delta p \Delta x \geq \hbar$$

– the Heisenberg indeterminacy or (unfortunately termed) "uncertainty" principle. (The rigorous result, with Δp and Δx suitably defined, is $\Delta p \Delta x \geq \frac{\hbar}{2}$).

How to interpret the indeterminacy principle? One tempting possibility, popularized (unfortunately) by Heisenberg in his 1930 Gifford lectures, is to interpret it as saying: An electron "really has" simultaneously an exact position x and exact momentum p, but we can never simultaneously ascertain them with arbitrary accuracy (hence, "uncertainty"). Heisenberg's original argument went roughly as follows: imagine that to determine the

*Note that the symbol Δx is now being used in a sense that is standard in the QM literature, but is somewhat different from its usage in earlier lectures. Since the context is quite different, it is hoped that this will not cause confusion.

position of the particle you shine on it a light beam with wavelength λ. It is a standard result of classical optics that to resolve the position within Δx we need λ to be *at most* of order $2\pi\Delta x$. However, the light beam actually consists of photons with momentum $p = \frac{h}{\lambda}$; in colliding with the electron, they will transfer some momentum which is of order p *and is unknown.* Consequently, after the observation, the electron momentum will itself be uncertain by an amount Δp which is of order $\frac{h}{\lambda}$. Since we saw that $\lambda \leq 2\pi\Delta x$, it follows that $\Delta p\Delta x \geq \hbar$, as previously obtained.

This argument, although tempting and of some historical importance, is actually quite misleading. In the first place, it is clear that we have "proved" the indeterminacy principle for an electron only by, in effect, already assuming it for the photon (in that we assume the de Broglie relation $\lambda = \frac{h}{p}$ for the latter). So at best, the indeterminacy principle demonstrates the "holistic" property of quantum mechanics – to deny the principle for one kind of particle would allow the possibility of denying it for all. However, a more serious objection is that it gives impression that the characteristic anomalies of the behavior of particles predicted by quantum mechanics and observed experimentally are due to some kind of disturbance by the measuring apparatus. As we shall see in a subsequent lecture (20), this point of view is very difficult to maintain in the case of certain experiments involving pairs of particles that are spatially separated.

Thus, it is necessary to interpret the indeterminacy principle not as the thesis that you cannot *measure* (determine) the momentum and position of a particle simultaneously with arbitrary accuracy ("uncertainty"), but rather that a particle cannot simultaneously *possess* values of these quantities that are definite to arbitrary accuracy (indeterminacy). An electron (or photon) just *is not the kind of thing* that can possess an exact position and an exact momentum simultaneously! An alternative, more operational, formulation is that it is impossible to prepare a single "ensemble" of electrons such that measurements on one subset of them will show a unique value of x while measurements on a different subset show a unique value of p.

In any case, it is immediately clear that the indeterminacy principle means that we can no longer carry out one program with which we are very familiar in classical particle mechanics – namely, to work out the trajectory of a particle (e.g., a cannonball) from a knowledge of its initial position and velocity $\left(\frac{\text{momentum}}{\text{mass}}\right)$. Why doesn't this matter in practice for things like cannonballs (believed also, in principle, to be described by quantum mechanics)? Answer: what matters for the "Newtonian" program is the simultaneous accuracy of definition of position and velocity, not position and momentum. Since $v = \frac{p}{M}$, we have

$$\Delta x \Delta v \approx \frac{\hbar}{M}$$

and this indeterminacy product is very small for large M.†

This is a good point at which to raise the question: why, in everyday life, is it natural to regard the electron as a "particle" and light as a "wave", despite the fact

†But it can still have surprisingly large effects: compare the "pencil balancing" problem in the presence of quantum mechanical effects.

that quantum mechanics says that both have both aspects? Actually, there are several relevant points:

1. Because of the nature of electrons,[‡] it is impossible to produce a large amplitude ("classical") electron wave.
2. By contrast, many electromagnetic sources contain vast numbers of photons, which are therefore very difficult to see individually.
3. For electrons of energy, say, of the order of kilovolts (as in a TV tube), the de Broglie wavelength, and hence appropriate diffraction gratings, are extremely small (the "wave" aspect is hard to see); by contrast, for light, the momentum change Δp in collisions is extremely small (the "particle" aspect is hard to see).

Let's now return to the "2-slit" experiment as such and look at it from rather more general point of view. The crucial observation is that the electrons arrive at the final (scintillating) screen as (apparently) discrete particles, yet we find that in general:

$$N_{a+b}(x) \neq N_a(x) + N_b(x)$$

As we saw in the last lecture, quantum mechanics explains this as due to the fact that

$$\psi_{a+b}(x) = \psi_a(x) + \psi_b(x)$$

and thus

$$N_{a+b}(x) = (\psi_{a+b}(x))^2 = N_a(x) + N_b(x) + 2\psi_a(x)\psi_b(x)$$

i.e., it is due to the "interference" term. Now the crucial point is that the interference term involves both ψ_a and ψ_b simultaneously. In other words it looks very much as if "something" came through a slit A and "something" through slit B, for each electron that came through the apparatus.[§] Now, of course, if the electron were really a wave this causes no problem, because it is the nature of a wave that it has non-zero disturbances at different point in space simultaneously; but this is precisely what the electron does *not* seem to do on the final screen.

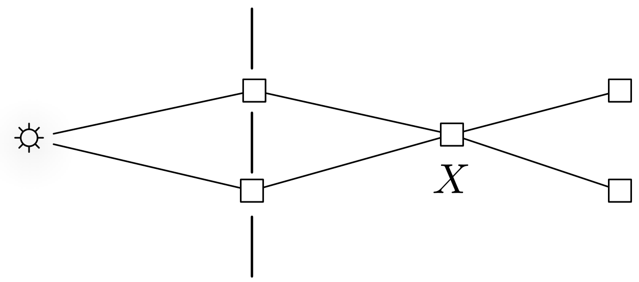
So an obvious question arises: Could we not put a detector (e.g., a flashlight) in front of each of the slits, and determine whether a given electron did indeed come through one or the other or somehow through both while still allowing it to propagate to the final screen? In principle, yes. In fact this precise experiment has for various practical reasons not been done in exactly this form in the two-slit context. However, enough experiments which are very close in concept to it have been done that one is pretty confident what the answer would be: each electron is indeed seen to come through one slit *or* the other, never "both" and never "neither" (this is verified directly). However, under these conditions the statistics of arrival of electrons on the far screen shows no interference – we recover the "naïve" prediction:

$$N_{a+b}(x) = N_a(x) + N_b(x)! \qquad \text{(no interference)}$$

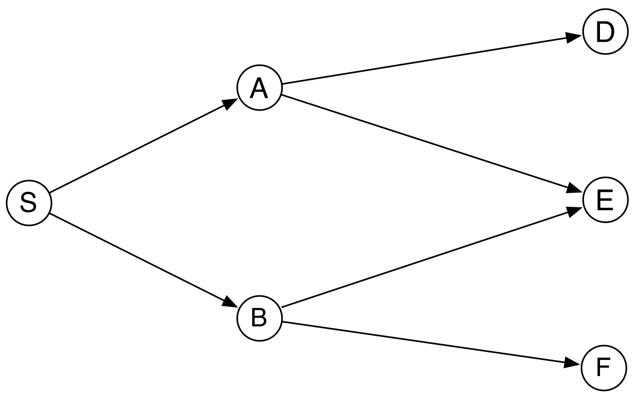
[‡]By the so-called "Pauli principle", no two electrons can occupy the same state.

[§]Note in particular the implications of destructive interference.

It is tempting to argue that the correct interpretation is that yes, each electron did indeed come through just one of the slits, and that the destruction of the interference pattern is a result of an unavoidable disturbance by whatever device (e.g., light) we use. As we have seen, this was Heisenberg's original application of the indeterminacy principle. But this won't work, not only because of the arguments already given, but also because of the possibility of "delayed choice" experiments (cf. Sklar pp. 166-7), in which the choice of whether to measure the interference pattern at X, or rather "which way?", is postponed to long after the slit- transit process.



The 2-slit experiment is actually a special case of a very much more general case in quantum mechanics. Consider the following general setup:



Here there is some "ensemble" of entities that can proceed from a "source" S to one (or both!) of two states A, B, and hence to D, E, or F. In particular, paths are (or may be) open to E through both A and B. The "states" A, B, etc., need not correspond to spatially distinct positions: they could, for example, correspond to different internal states of various elementary particles. We can imagine for convenience that we have a supplementary "counter" telling us when a system of the ensemble has left S (an "event-ready" detector). We can also imagine counters installed at D, E, F (but *not*, for the moment, at A or B). The salient points (confirmed, at least circumstantially, in a host of experiments) are these:

1. Whenever a system is detected as leaving the source S by the event-ready detector, it is always found to arrive in exactly one of D, E, F.

2. If the number (or fraction, or probability) of systems reaching E is measured when only path A is open, only path B is open, and both paths are open, the corresponding numbers are related by

$$N_{a+b}(E) \neq N_a(E) + N_b(E)$$

showing the effects of interference of the two paths.

3. Nevertheless, if measuring devices are set up at A and B, we always find that each particular system is found either at A or at B (never both, never neither). However, under these circumstances there is no interference observed at E.

Thus we appear to have found the following extremely puzzling state of affairs. Whenever we arrange to observe which path, A or B, is followed by an individual system, we always seem to get a definite result. Yet when we do not observe this, the statistical behavior of the ensemble as a whole seems to imply that each system did not follow one path or the other! (Note, by the way, that although the phenomenon of interference refers to the histograms – i.e., to the statistical behavior of the ensemble as a whole – it seems that there is at least one statement that one can make with 100% confidence on its basis about the individual members, namely: no system of the ensemble arrives at a point of total destructive interference).

The Copenhagen interpretation

The name "Copenhagen interpretation" (or perhaps better, "Copenhagen non-interpretation") is given to a collection of recipes for coping with the conceptual problems of quantum mechanics that evolved, in Copenhagen and elsewhere, in the late 20s and early 30s at the hands of Bohr, Heisenberg, Reichenbach, and others, and for many years was the "establishment" approach to these problems. (It is to be noted, however, that a number of distinguished physicists [Einstein, Schrödinger, Von Neumann...], including some of the founding fathers of quantum mechanics, never embraced the Copenhagen interpretation and indeed in some cases publicly opposed it). Bohr and Heisenberg often try to give, in their writings, the impression that the Copenhagen interpretation is an inexorable consequence of a particular kind of empiricist philosophy of science. However, it has been frequently claimed, perhaps with some justice, that this is a justification after the event, and that the original genesis of the Copenhagen interpretation was for largely practical reasons and had little to do with any philosophical viewpoint. The version given by Bohr on the one hand and Heisenberg on the other (and for that matter the versions given by each of them at different times) differ somewhat in detail but are very similar in general spirit. Here, I will concentrate for definiteness on Bohr's version as given, e.g., in his collection of essays *Atomic Physics and Human Knowledge.*

Bohr first makes the (obviously true) observation that all our knowledge of events at the atomic level is at the last resort based on the effects of the atomic system on *macroscopic* measuring apparatus.¶ He then formulates two major principles:

¶Examples: particle counters, photographic plates, scintillating screens... The human eye may be

1. It is necessary to describe the preparation, working and output of the macroscopic measuring apparatus in the language of classical physics. (Note: not Newtonian mechanics, nor Maxwell's equations, etc.; but rather, the language of "did click" or "did not click", "read 5A" or "read 10A", etc.) Why? Bohr's answer is simply: "because we must" if we are to communicate with one another at all.

2. Microscopic objects (such as electrons or photons) should not be thought of as even possessing properties in the absence of a specification of the precise *macroscopic* measuring apparatus which is make measurements on them. Thus, for Bohr, an electron does not possess any properties in its own right, but rather is a link or *relation* between the preparation and macroscopic measurement apparatus. It is not a question of *disturbance* of existing properties by the measurement process, rather that such properties did not exist at all until they were "actualized" by that process.

Here is P. K. Feyerabend, paraphrasing what he believes to be Bohr's view:

> [The QM state is a relation between (micro)systems and (macro) measuring devices.] A system does not possess any properties over and above those that are derivable from its state description (the completeness assumption). This being the case, it is not possible, even conceptually, to speak of an interaction between the measuring instrument and the system investigated. The logical error committed by such a manner of speaking would be similar to the error committed by a person who wanted to explain changes of velocity of an object created by a transition to a different reference system as the result of an interaction between the object and the reference system.

Bohr then goes on to formulate the idea of complementarity: in a given physical situation we may have to make a choice to what it is we wish to measure. A good example is the "delayed-choice" thought-experiment described above: we can either decide to measure the interference, which requires choosing to detect at a place where the two beams are coincident, or to make a "which way" (*welcher Weg*?) measurement, which requires measurements at points where only one beam is to be found. But note that this argument, while not necessarily circular, is at least "holistic": compare Reichenbach's discussion of the proposal to observe interference and simultaneously measure "which way?" by measuring the recoil of the diaphragm through which the electron is diffracted. The only reason this won't work is that the diffraction diaphragm itself must satisfy the indeterminacy principle! (Compare the Heisenberg "gamma-ray microscope" thought experiment.)

Bohr himself obviously had a very high opinion of the value of the notion of "complementarity", and in his later years attempted to apply it to biology, psychology and other areas of human knowledge in a way in which practitioners in those fields have not always found useful.

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an apparent exception (as few as 8 photons are detectable), but note here the need for considerable amplification before conscious registration of the event.

A more philosophically-sophisticated version of the Copenhagen interpretation was given by the empiricist philosopher Hans Reichenbach in his book *Philosophical Foundations of Quantum Mechanics.* In the spirit of empiricist philosophy, Reichenbach makes a fairly sharp distinction between those features of the world that are directly observable, and those that are, in the last resort, a matter of interpretation. (Though in contrast to the more extreme adherents of the logical empiricist and logical positivist schools, he is willing to allow that the former category includes not only "sense-data" but also those features of the everyday world that we infer directly from them.) With regard to those features of the world that are not directly observed, he distinguishes between an "exhaustive" interpretation (as in "the tree is actually still there when I turn my back") and a "restrictive" interpretation ("the world behaves as if the tree were actually still there when I turn my back"). He also states two principles that define what he calls a "normal [interpretative] system" for a given class of object, namely

1. The laws of motion are the same whether the objects are observed or not.

2. The state of the objects is the same whether they are observed or not.

By an examination of the experiments that are used to establish the quantum-mechanical picture, Reichenbach then concludes that, while at the level of everyday life there is nothing to stop us holding an exhaustive interpretation that is "normal" in his sense, at the atomic level, we must either live with a restrictive interpretation (as Mach would have had us do) or violate one or both of the conditions 1 and 2: i.e., at the atomic level the world *does not tolerate a normal exhaustive interpretation.* This, in his view, is an interesting fact about the world that need not have been true but in fact is.

It should be noted that Reichenbach's version of the Copenhagen interpretation, like Bohr's, rests heavily on the notion of a sharp distinction between microscopic objects such as electron or photons, on the one hand, and the macroscopic (everyday-level) pieces of apparatus such as counters, voltmeters, etc., which we use to obtain information about them on the other. Given this distinction – which at least in Reichenbach's day must have seemed very natural – his version perhaps expresses most clearly the important fact that so long as we are content to talk only about the output of our macroscopic instruments, there is nothing particularly paradoxical about the experiments that are used to support the quantum mechanical world view; it is only when we follow our natural instinct to extrapolate beyond the raw data (i.e, formulate an "exhaustive" interpretation) that we seem to get into trouble.

What are the points of vulnerability of the Copenhagen interpretation? The first is perhaps more apparent than real, namely the tendency of its advocates to rely on dubious philosophy and even to claim, on (pseudo-)philosophical grounds, that the world *has* to be as quantum mechanics describes it.

> The defenders [of quantum theory] use bad and irrelevant arguments for a point of view with whose physical fertility they are well acquainted and of whose value they therefore have a very high opinion. The opponents, ignorant of the features of physical practice but well acquainted with the irrelevant

> descriptions of it, set out to destroy those irrelevant arguments and believe that they have thereby destroyed the point of view those arguments were supposed to support.
>
> (P.K. Feyerabend in R.G. Colodny, ed., *Frontiers of Science and Philosophy*, p. 202.)

A second point of vulnerability is the apparent dogmatism of the claim, made in at least some versions of the Copenhagen interpretation, that the "understanding" we have obtained of the microscopic world from quantum mechanics is the ultimate truth about it and that we shall never reach a deeper level of description. The history of physics certainly should induce some skepticism about any such claim, and an analogy often pointed to by opponents of the Copenhagen interpretation is the case of Brownian motion: originally thought to be a matter of absolute chance, this is now understood as being due to the very complicated, but in principle deterministic, motion of a host of molecules that bombard the Brownian particle. How can we be sure, critics of the Copenhagen interpretation ask, that we will not some day find just such a deterministic picture underlying the apparently random events in, say, a two-slit experiment, to which quantum mechanics as such allows us to ascribe no cause? In other words, how do we know that the real physical system is not described by extra "hidden" variables that are not allowed for in the quantum mechanical description? I will return to this question in lecture 19.

Finally, the aspect of the Copenhagen interpretation that in recent years has emerged as perhaps its most deep-rooted problem is the necessity of making a sharp distinction between the microscopic and macroscopic worlds, and in particular the fact that no account is given either of the precise point at which a "measurement" occurs, or of what actually happens in such a "measurement". I will discuss this problem in much greater detail in lectures 21 and 22.

A final quote about the two-slit experiment:

> The question now is, how does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon that I have given: that is, a description of it. They can give you a wider explanation, in the sense that they can do more examples to show how it is impossible to tell which hole the electron goes through and not at the same time destroy the interference pattern. They can give a wider class of experiments than just the two slit interference experiment. But that is just repeating the same thing to drive it in. It is not any deeper; it is only wider. The mathematics can be made more precise; you can mention that they are complex numbers instead of real numbers, and a couple of other minor points which have nothing to do with the main idea. But the deep mystery is what I have described, and no one can go any deeper today.
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> (R.P. Feynman, *The Character of Physical Law*, p. 145).