Physics 419: Lecture 3: de Brache, Kepler, and Galileo: The Birth of Modern Cosmology
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As was evident from the previous lecture, Coppernicus was a reluctant revolutionary. While he had one foot planted in helio-centrism, he had the other entombed in Platonism and Aristotelianism by his use of countless epicycles to account for the irregular motion of the planets. He offers no explanation for the physical origin of the epicycles. As a consequence, in the Coppernicun universe, the motion of the planets is a complete mystery. After Coppernicus, astronomy took a decisive turn to answer the fundamental questions of planetary motion. In this lecture, we will focus on the laws for planetary motion formulated by Kepler and the attempts by Galileo and Kepler to account for the forces that produce the observed orbits of the planets. While neither really arrived at the final synthesis and hence a complete explanation of planetary motion was still lacking upon their deaths, they came rather close to enunciating the phenomenon of gravitation formulated by Newton. From Newton, we have the first quantitative theory for the motion of the planets.

We first focus on Johannes Kepler. Kepler was a rather flamboyant, free-spiritied person who boasted he knew not only the exact time of his birth, 27 December at 2:30 pm, but also the exact time of his conception, 16 May 1571 at 4:30. He devised five different spellings of his name: Kepler, Keppler, Khepler, Kheppler, or Keplerus. All five spellings and the conception and birth times appear on a horoscope Kepler concocted for himself. While he became the first precise astronomer, he had a profound interest in astrology which did not wain even after he formulated the laws of planetary motion. Whichever spelling you decide on, you should use it throughout your papers. In astronomy, Kepler set out to answer two questions: 1) why are their only six planets (at the time of Coppernicus, there were only six planets) instead of 30 or five-hundred and 2) what determines the distances and velocities of the planets as they orbit the sun. As is evident, the questions Kepler asked are fundamentally different from the rather qualitative question Coppernicus attempted to answer, namely 'does the sun go around the earth or does the earth go around the sun.' While his inquiry into the first question produced nothing of any value, his investigations on the latter produced the first natural laws. However, Kepler saw an intimate connection between the two questions. As we will see, his answers to these two questions reflects the schizophrenia of cosmologists of his time that allowed them on the one hand to denounce the Platonic universe, but at the same time retain some semblance of it on the other.

1 Platonic Solids and the 6 Planets:

The key step which led to resolution of the first question was a geometrical construction in which a circle was enscribed in a triangle and an outer circle was circumscribed around the triangle. Kepler noticed that by suitably adjusting the lengths of the triangle, he could reproduce the ratio of the orbits of Saturn and Jupiter. Saturn and Jupiter are the two outermost planets. A triangle is the simplest figure in geometry. He then went on to place a square between Jupiter and Mars, a pentagon between Mars and Earth, etc. While this scheme worked, it did not terminate in any logically necessary way. It then occurred to Kepler that if he used three-dimensional objects, such as the Platonic solids, then the series would have a definite end. Platonic solids are perfect 3-dimensional solids with identical faces. As such there can not be an infinite number of such solids. In fact, their are only five: 1) the tetrahedron (pyramid), 2) the cube, 3) the octahedron (eight equilateral triangles), 4) the dodecahedron (twelve pentagons), and 5) the icosahedron (20 equilateral triangles). As there are six planets, there can only be five solids that can fit perfectly between the six planets. These solids must be the Platonic solids since the universe is perfect. This is the line of reasoning that led Kepler to his resolution of his question ‘why do only 6 planets exist?’ Throughout his life Kepler believed this to be self-evident. He firmly believed that Platonic solids had a fundamental design in the universe. In fact, when Galileo reported the discovery of 4 additional planets in 1610, Kepler immediately dismissed them as satellites of some existing planet. This turned out to fairly close to the truth. In actuality, Galileo had discovered the four moons of Jupiter. Kepler published his Platonic solid work in a book entitled *Mysterium Cosmographicum.* He was 25 and had all the confidence now to tackle his second problem, the origin of planetary motion.

While there is nothing true in what Kepler had done, it does serve to illustrate that one cannot underestimate the importance of confidence building in the scientific process. Had he felt defeated on this problem, he might have abandoned altogether his quest to really do something serious. The Platonic solid example of Kepler underscores the fact that scientific confidence need not necessarily arise from a solid piece of work. All that is necessary is the perceived belief that the ultimate secrets of nature are not beyond your grasp. Kepler is unique in that his story appears to be the only one in history where delusion led to real physical laws. A realistic question to ponder here is, why are we capable of such delusion but yet at the same time such clarity of thought? Perhaps the answer to this mental schizophrenia lies in psychology rather than philosophy.

2 Precursor to Gravity

Kepler then turned to the concrete problem of the relationship between a planet’s distance from the sun and the length of its year. His desire was to formulate a mathematical relation between the two. From ancient times, the exact periods of all the 6 planets was known: 3 months for Mercury, 7.5 months for Venus, 1 year for the earth, 2 years for Mars, 12 years for Jupiter, and 30 years for Saturn. Uranus, Neptune, and Pluto had not been discovered yet. So there is an obvious correlation between distance from the sun and the length of the planetary year. The further a planet is from the sun the longer it takes to revolve around the sun. But what is the precise relationship? Saturn is twice as far out in space as is Jupiter. However, the year on Saturn is 30 years as opposed to 24 years. From this Kepler reasoned that the speed of the planets must be decreasing as their distance from the sun increases. At this point, Kepler made a step not made previously: he deduced that there must be some force emanating from the sun which diminishes in magnitude the further a planet is from the sun. It is this force that drives the planets in their respective orbits around the sun. This step represents a true milestone in the construction of a cosmological theory of the universe because it was the first time physical causation was attributed to the sun. While Kepler did not formulate the principle of gravitation, he did know that the gravitational force must decrease with increasing distance. It was Newton who made this step. But nonetheless, once Kepler realised that a force must exist that governs planetary motion, he was convinced that planetary motion must be describable in mathematical terms. At this point he scoured the available astronomical data to see if he could decipher any reproducible patterns in planetary motion. This was a painstaking process which Kepler thought would take only 8 days. It was 8 years later that he formulated his first law.

3 Kepler's Laws of Planetary Motion

Kepler's formulation of the laws of planetary motion demonstrates beautifully how hypotheses about physical phenomenon evolve from the realm of the purely plausible to the realm of law. Physical laws are precise verifiable or falsifiable statements about nature. Physical statements are elevated to the realm of law when no (reasonable) exceptions exist to what they purport to be true. More on this later! As such the process of finding the laws of nature is one which necessarily involves experimentation. No physical law can be established without experiment. Physical laws capture the way the universe is. They represent the rules by which nature lives.

Consider now the case of Kepler. He needed a vast amount of experimental data to decipher precisely how the planets move. The astronomer with the largest collection of data on the precise orbits of the planets was Tycho de Brache. de Brache invited Kepler to take up a research position in his observatory in Benatek, a city close to Prague. Kepler's job was to solve the Mars problem: why does the motion of Mars deviate so strikingly form a perfect circle. Until Kepler, astronomers invented epicycle upon epicycle to explain such deviations in the orbits of the planets. By the time that Kepler arrived in Benatek, 1600 A.C.E., Tycho and his assistants were convinced that Mars posed a serious problem to the standard planetary cosmology. No amount of epicycles could explain the Martian orbit.

3.1 Three Preliminary Innovations

Three innovations highlighted Kepler's path to the laws of planetary motion. Not knowing that the orbit was actually an ellipse, Kepler reasoned that the while the planets turn around the sun, the center of their orbit must not be the sun. He then constructed a circle and placed the sun slightly off center. He then posed the question: If the force that moves the planets comes from the sun, why do the planets insist on turning around the center rather than the sun. The correct answer is that there must be two forces: one from the sun and the other from the planet itself. The tug-of-war between these two forces could give rise to motion about the center of the circle as opposed to the sun. The force arising from the planet is known as the inertial force. So before he found the first planetary law, Kepler had already intuited gravity and inertia. This is truly remarkable. We will say more about this next lecture when we consider the Newtonian synthesis.

The second innovation was to debunk the age-old view that the planes of the orbits of the various planets varied wildly from planet to planet. To explain many of the irregularities of Mars, Coppernicus as did Ptlomey before him reasoned that possibly the plane of the Mars orbit oscillates in space. Using the extensive observational data of de Brahe, Kepler showed that the angle between the planes of Mars and Earth remained always the same, $1^o50'$. Further observations revealed that in fact all the planets moved in essentially the same plane.

His third innovation is most daring: the planets do not move at constant speed along the orbit. From Plato to Coppernicus, a given in cosmology was constant speed along perfect circles. Since speed is inversely related to distance from the sun, this latter innovation already implies that the sun-planet distance does not remain constant along the planetary orbit. So from this, if one places the sun at the center of the orbit, one is forced to consider a non-circular orbit. But this is not what Kepler did at this point. He still stuck with the circle and proceeded to determine the relationship between the speed of a planet and the arc length the planet sweeps in a given period of time. The result of this study is contained in the 16th Chapter of his book *New Astronomy*.

3.2 The Problem of Mars

However, circular motion was soon to go out the window. Tycho had recorded four different positions of Mars at different dates when the planet was in opposition to the sun. With these four positions, one could solve a rather complicated geometry problem to find the radius of the orbit, the direction of the orbit and position of central point of the orbit. Kepler showed that a circular model produced errors for the actual position of Mars that were off by 8 minutes arc. This corresponds to a distance on the order of the radius of the moon. This error was too large to be ignored. However, the actual geometric shape was not forthcoming for quite some time. What he did next was to divide the orbit of the earth into 360 parts and compute the actual distance of each part of the orbit from the sun. To determine the time a planet takes to traverse from 0^o to 90^o requires adding up the contribution from the first 90 compartments. Let us shade this region. Kepler noticed that the area (as measured by the line connecting the planet and the sun) swept out by a planet is related to the time it takes a planet to travel a certain distance on its orbit. Regardless of the arc length of the orbit, as long as the area swept out is the same, the time it takes a planet to cover that arc length must be equal. This is Keppler's second law:

Second Law: Consider the area swept over by a line connecting a planet to the sun. This line will sweep out equal areas in equal times.

Kepler realized that the procedure he had used to calculate the area was not entirely valid; however, he reasoned that the error introduced by using a circle would just about cancel the first error. But this is actually not true. The determination of the actual orbit took another two years. His first guess was an oval then an egg. But he returned to Tycho's data and plotted the positions of Mars at different positions in its orbit and tried to determine which geometrical shape they described. To determine a circle, three points are needed, a straight line just two points. Kepler had four. It took him two years to solve this geometry problem but when he did solve it, he did not realize precisely what mathematical object it was. He then computed the orbit as if it were elliptical and the results fell exactly on the curve he had constructed from the data.

First Law Planets move in elliptical orbits with the sun at the focus of the ellipse.

Kepler was quite unhappy with this law because of the lack of aesthetic appeal of an ellipse. He referred to his ellipse as "a cart-full of dung." The formal equation for an ellipse is $R = 1 + e\cos\theta$, where R is the distance from the sun, θ the longitude referred to the center of the orbit and e the eccentricity.

Third Law:

The period (T), defined as the time it takes a planet to complete one complete cycle around the sun, is related to radius (R) of its orbit as follows: T^2=constant R^3.

It is in this law that Newton's law of universal gravitation is hidden. In fact all three laws fall out naturally from the single fundamental principle of universal gravitation.

4 Awaited Synthesis

What was absent from the Keplerian universe was an understanding of why is it that the planets move in elliptical orbits and why is it that their periods and distance from the sun are related in such an odd fashion. As mentioned earlier Kepler reasoned that forces from the sun and the planets must lie at the heart of planetary motion. According to Kepler, the planets were driven around in their orbits by spokes of a force originating from the sun. Enter Galileo. Galileo rejected the elliptical orbit hypothesis although it was shown to explain the experimental data and held onto the circular orbits with all of their epicycles until his death. Galileo purported there is no need for any force to act on a planet because circular motion is self-perpetuating. That is, for Galileo, an object with no external forces acting on it will proceed to move in a circle. For Kepler, it was the inertia, the laziness of a planet, that made it lag behind. The confusion was heightened by Descartes. Descartes argued that an object with no forces acting on it will proceed to move in a straight line. It turns out this is quite close to the truth. For Descartes, however, uniform motion and rest were distinct qualities of an object. Then if this is true, why do the planets move in an ellipse. Descartes reasoned that the planets are whirled around by vortices in an all-pervading ether. Hence, there was complete disagreement on the two fundamental points: 1) on the nature of the forces that drives planets in their orbits and 2) on the ultimate fate of an object that is left alone with no external forces acting on it. What was missing was the concept of gravity and the laws that govern forces. It was this missing link that Newton provided.

For the Record here are Descartes' laws of motion:

Law 1. Each thing, in so far as it is simple and undivided, always remains in the same state, as far as it can, and never changes except as a result of external causes... Hence we must conclude that what is in motion always, so far as it can, continues to move. (Principles Part II, art. 37)

Law 2. Every piece of matter, considered in itself, always tends to continue moving, not in any oblique path but only in a straight line. (Principles Part II, art. 39)

Law 3. When a moving body collides with another, if its power of continuing in a straight line is less than the resistance of the other body, it is deflected so that, while the quantity of motion is retained, the direction is altered; but if its power of continuing is greater than the resistance of the other body, it carries that body along with it, and loses a quantity of motion equal to that which it imparts to the other body. (Principles Part II, art. 40)

The first three laws have to do with persistence or power while the third to reconciliation of the motion of bodies subject to motion described Laws 1 and 2.

We will contrast these with Newton's laws next class.