

The quantum world is stranger than we can imagine

Bell's theorem (1964)

Limits on hidden-variable accounts

*"Bell's theorem is the most profound
discovery of science. Henry Stapp*

A story

Say that you want to find out why people like pepperoni, mushrooms, and olives on pizza.

- You ask many people, and they give Yes/No answers when asked about each. (Say 50% yes for each.) But you don't notice any distinguishing properties of the people who say Yes.
- Does that mean you can conclude that the answer is random, some sort of momentary glitch that occurs in a person's response?
- Of course not- you may just have missed the "hidden variables" needed to understand taste. There MAY be something in each person's mind ahead of time that determines their answer, or maybe not.

Ask couples

- Each couple gives the same answer to the one question you ask them, so you know there was already something in their heads that determined the answer. Otherwise how could they all get their stories straight? So there WAS some hidden variable determining the outcome.
 - A slight complication- all these folks seem to get confused after one question. If you ask each member of the couples a second question, the perfect correlation between them is lost. So you're only allowed to ask one of each person.
- Now you try asking one person about pepperoni, and the other about mushrooms. You find that 85% of the time, they give the same answer (YY or NN). So you conclude that whatever makes people like mushrooms usually makes them like pepperoni.
- You try the same thing asking about mushrooms and olives. Again 85% of the time, they agree. (YY or NN)
- Now you ask about pepperoni and olives. What extent of agreement do you expect? The 15% who had different opinions on M-P and the 15% with different opinions on M-O might be the same people. Then there would be 0% disagreement on P-O. Or maybe they're all different people. Then there would be $15\% + 15\% = 30\%$ disagreement on P-O. Or it could come out anywhere in between.

Whoops

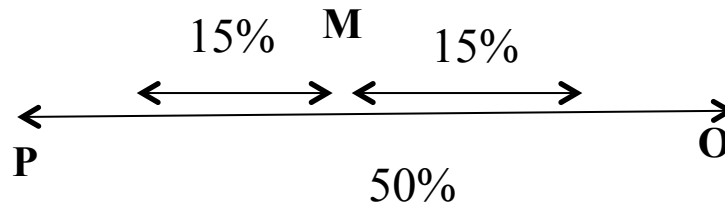
- You now ask the couples do you like pepperoni/olives?
 - **they give the same answer 50% of the time.**
- What gives?
 - Maybe there was a statistical fluke. You try the same thing again with 100,000 couples- and get the same result.
 - Maybe the couples were secretly signaling each other, getting their stories coordinated (on some questions) AFTER the questions were asked.
- Ask the questions in sealed boxes. Same result.
- Ask the questions SIMULTANEOUSLY (in Earth frame). Same result.

Maybe the couples were getting their stories straight only on the questions which they knew they would be asked.

- You draw the questions out of hats, in the sealed boxes, simultaneously. And get the same result.

Science Fiction?

- Conclusion: this story is obviously fiction. The identical results on any ONE question tell you that (whether or not you believe that people in general are spontaneous and random) on these particular issues there had to be some prior content in their heads determining the answers. But there is NO WAY of assigning answers to the 3 questions (M,O,P) which gives these results for the disagreements:



Tell the same story about particles

- "couples" are pairs emitted together in a decay process.
- "Like pepperoni?" becomes "spin up along x?"
- "Like olives?" becomes "spin up along y?"
- "Like mushrooms?" becomes "spin up along direction halfway between x and y?"

Experimentally the results are as we have described.

- Most of the real experiments use a slightly different set-up with photon polarization, with the same logical structure.
- Obviously when we concluded that this story is science fiction we assumed something that's false.
- What did we assume about reality?
 - we assumed nothing whatever about QM-
 - » we didn't even mention it.
 - » It happens that QM precisely predicts the results,

Proof of Bell's theorem

- Assume there are 3 settings for the left and right detector.
- Quantum mechanics predicts:
 1. If R and L detectors are set the same detectors, always agree
 2. If we average over all 9 settings, we get equal number of agreements and disagreements.

These facts rule out a local hidden variable theory.

PROOF: assume each particle carries along information that tells what will happen for the 3 settings.

- There are 8 possible hidden variables (yyy,yyn,yny,...nnn)
 - Call the unanimous variables (yyy or nnn) case A,
 - Call The mixed variables (yyn,yny,...) case B.
- For A particles, what is average over all 9 settings ?
- For B particles, what is average over all 9 settings?

Proof of Bell's theorem

- Assume there are 3 settings for the left and right detector.
- Quantum mechanics predicts:
 1. If R and L detectors are set the same detectors, always agree
 2. If we average over all 9 settings, we get equal number of agreements and disagreements.

These facts rule out a local hidden variable theory.

PROOF: assume each particle carries along information that tells what will happen for the 3 settings.

- There are 8 possible hidden variables (yyy,yyn,yny,...nnn)
 - Call the unanimous variables (yyy or nnn) case A,
 - Call The mixed variables (yyn,yny,...) case B.
- For A particles, what is average over all 9 settings ? (100%)
- For B particles, what is average over all 9 settings? (55.5%)

How can we choose A or B to satisfy #2?

We assumed

- Realism. Not that the world must be deterministic, but only that those aspects of it which can be predicted perfectly are determined by some element of reality. You can predict the result on any particle by making the corresponding measurement on its partner.
 - "If, without in any way disturbing a system, we can predict with certainty (*i.e.*, probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." (Einstein, Podolsky, Rosen, 1935)
- Local causality. The value of a property possessed by an isolated system cannot be affected by any operations carried out at a sufficient (*i.e.*, spacelike) separation from it.
- Induction. (no conspiracies) The unmeasured values (chosen by random quantum processes) are not statistically different from the measured values. The properties of an ensemble are defined completely by the preparation conditions. In particular, the distribution of "possessed values" of a variable for the subensemble which we actually measure is identical to the distribution for the complete ensemble. (random sampling)

Which one are you willing to give up?

Loopholes?

1. The initial passage of the particles through the angular momentum selectors are space-like separated. In the first generation of experiments, the conversion of those micro-events to some large-scale device setting was slow enough for it to be conceivable that a time-like signal could propagate between the detectors before "measurement" is complete. This loophole is now closed in some experiments, e.g. with satellites many km apart.
2. Detection efficiency. There's a complication in that many particles are missed, requiring some extrapolation. This is now closed in some experiments using atoms rather than photons.

No experiment yet reported quite closes all the loopholes simultaneously. Still, it's a long-shot to think that these loopholes offer any escape.

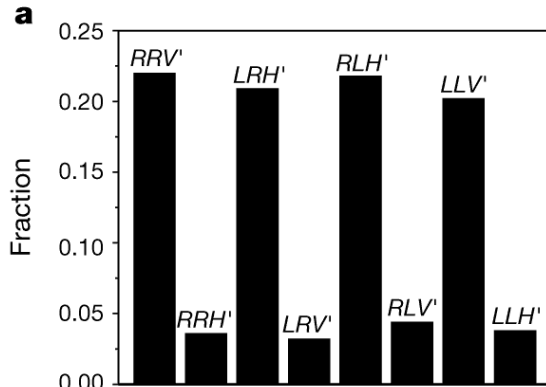
Does the problem lie with probabilities?

- We concentrated on probabilities and correlations. This may give the incorrect impression that the fundamental issue is the probabilities. It is possible to devise a 3-particle spin measurement in which **the distinction between QM and LR can be seen in every single measurement.**
- Prepare a 3-particle (each spin 1/2) state with the z-components of the spins described by $(uuu - ddd)/2^{1/2}$. Measure the x-components of the spins.
 - QM predicts $s_1 s_2 s_3 = -1$, always,
 - LR predicts $s_1 s_2 s_3 = +1$, always.
- A single measurement of the 3 spins could do the job. (See article by N. D. Mermin in *Physics Today*, June 1990.)

The new parable

- You can ask one question each of Anne,(1) Belinda (2), or Cary (3). The questions can be "olives?" (X) or "Onions?" (Y) . The answers can be "yes" (x=+1, y=+1) or "no" (x=-1, y=-1).
- You always find that if you ask one "olives?" and two "onions?" the product of the answers is +1. So it sounds as if each answer is determined by an "element of reality" since it is predictable by asking two OTHER questions.
- $X_1 Y_2 Y_3 = Y_1 X_2 Y_3 = Y_1 Y_2 X_3 = 1$
- So if those are just numbers we're talking about:
- $X_1 Y_2 Y_3 Y_1 X_2 Y_3 Y_1 Y_2 X_3 = X_1 X_2 X_3 Y_1^2 Y_2^2 Y_3^2 = X_1 X_2 X_3 = 1$ ALWAYS
- BUT QM says something strange: $X_1 X_2 X_3 = -1$ ALWAYS
- Essentially this experiment (using photons rather than spin=1/2) has been done.

Three-particle results



Nature **403**, 515-519 (3 February 2000) | doi:10.1038/35000514

Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement

Jian-Wei Pan¹, Dik Bouwmeester², Matthew Daniell¹, Harald Weinfurter³ & Anton Zeilinger¹

More than 85% of the time $X_1X_2X_3 = -1$ was found, which is impossible classically. The remaining 15% or so is attributed to imperfections in the analyzers, etc.

