The new invariants

Energy and momentum in relativity

Electric and Magnetic fields

What does “mass” mean?

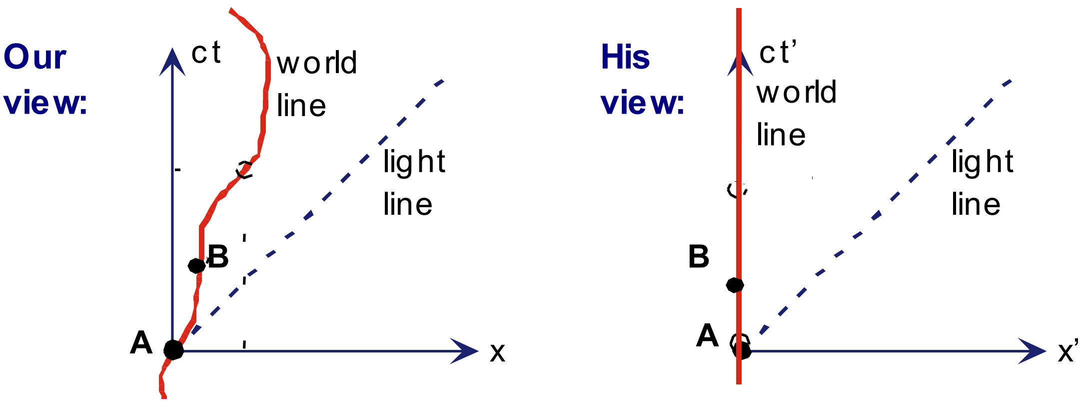
QUIZ at 1:20

4-dimensional physics

- The principle of relativity requires that if the laws of physics are to be the same in every inertial reference frame, the quantities on both sides of an = sign must undergo the same Lorentz transformation so they stay equal. <u>You cannot make any invariant from space or time variables alone</u>. That's why we call the SR world 4-D, and call the old world 3-D + time. No true feature of the world itself is representable in the 3 spatial dimensions or the 1 time dimension separately.
- In Newtonian physics, **p**=m**v** (**bold** means vector). Momentum and velocity are vectors, and mass is a scalar (invariant) under 3-d rotations. This equation is valid even when we rotate our coordinates, because both sides of the equation are vectors.
- The new "momentum" is a 4-d vector (4-vector for short). It's fourth component is E/c, the energy.
  - The factor of c is needed to give it the same units as momentum.
- The lengths of 3-vectors remain unchanged under rotations. So does the invariant "length" of 4-vectors under *Lorentz transformations*. The length^{2} of a 4-vector is the square of its "time" component minus the square of its space component: $(E/c)^2 - p^2 = (m_0c^2)^2$

4-D geometry

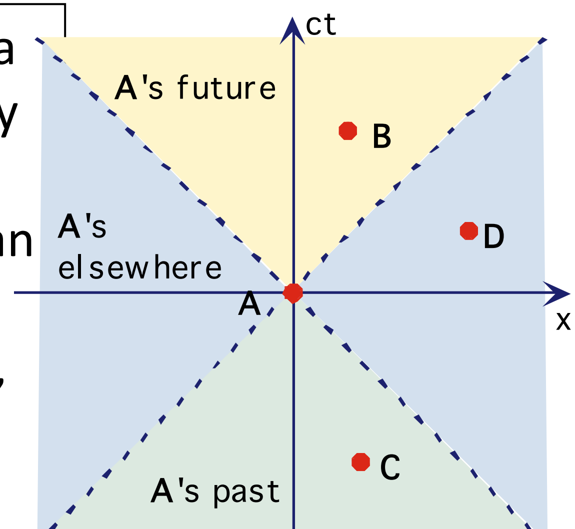
- In the geometrical interpretation of SR, c is just a *conversion factor*, the number of meters per second. The geometrical interpretation of SR helped lead Einstein to general relativity, although it didn't directly change the physics.
- World lines A graph of an object's position versus time:



If an object is at rest in any inertial reference frame, its speed is less than c in every reference frame. The speed limit divides the spacetime diagram into causally distinct regions.

Lorentz_transform_of_world_line.gif

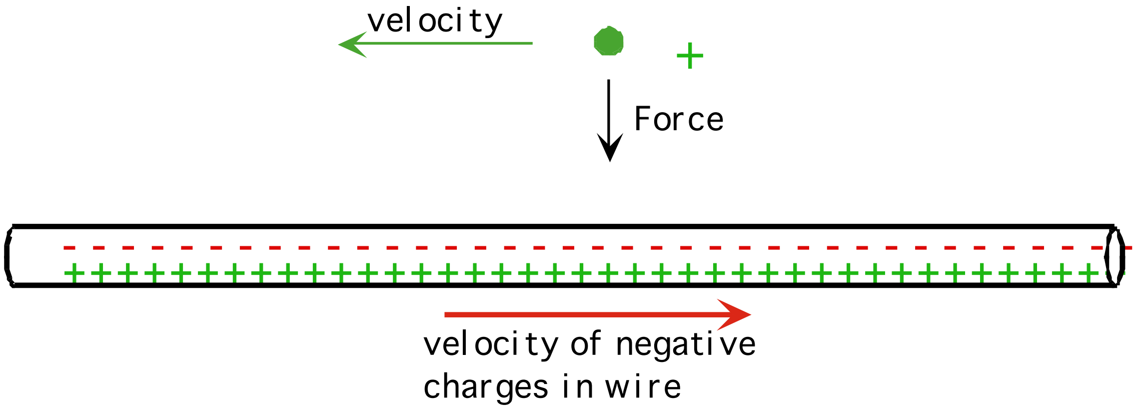
A, **B**, **C**, & D are events. **A** might be a cause of **B**, since effects produced by **A** can propagate to **B**. They cannot get to **D** without travelling faster than light, nor to **C** because it occurs before **A**. **C** might be a cause of **A**, **B**, and/or **D**. **D** could be a cause of **B**, since light can get from **D** to **B**.



If the interval, $(ct)^2 - x^2$, between pairs of events is positive ("timelike"), then a causal connection is possible. If it is negative ("spacelike"), then not.

Unification of electricity and magnetism

Einstein's one simple postulate solves a lot of problems. Consider the magnetic force on a moving charge due to the electric current in an electrically neutral wire (no electric field):



The magnetic force occurs when the charge is moving. If we look at it from the charge's point of view (*i.e.*, in its own "rest frame"), there can't be a magnetic force on it, but there must be some kind of force, because the charge is accelerating.

So, the principle of relativity tells us that the charge must see an electric field in its rest frame. (Why must it be an electric field?) How can that be? The answer comes from Lorentz contraction. The distances between the + and - charges in the wire are Lorentz contracted by different amounts because they have different velocities. The wire appears to have an electrical charge density. (The net charge in a current loop will still be zero, but the opposite charge is found on the distant part of the loop, where the current flows the opposite direction.)

Relativity of Fields

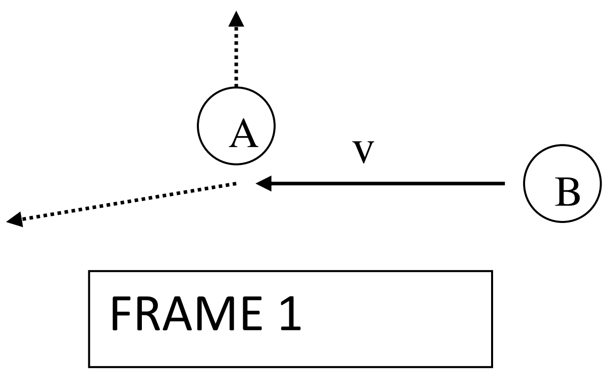
- When we change reference frames, electric fields partially become magnetic fields, and vice versa. Thus, they are merely different manifestations of the same phenomenon, called electromagnetism.
- The first oddity of Maxwell's equations was that the magnetic force existed between <u>moving</u> charges. But now we say that there's no absolute definition of <u>moving</u>.
- The resolution is that whether the force between two objects is <u>called</u> electric or magnetic is also not invariant.

Relativity is a Law

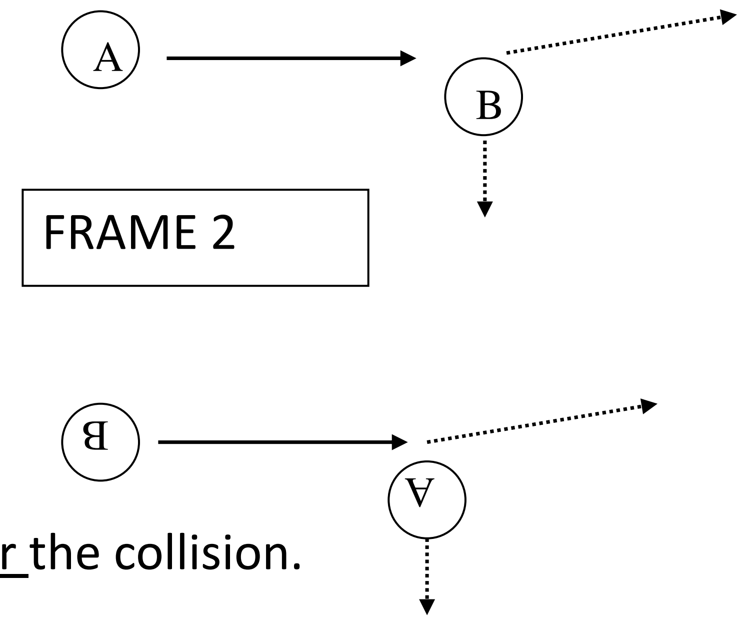
- Relativity might have sounded like some vague "everything goes" claim initially- at least in the popular press. Now we are deriving specific new physical laws from it.
- Relativity is a *constraint* on the physical laws. It says "No future physical law will be found which takes on different forms in different inertial frames."
- And “future laws” in 1905 include all the laws concerning nuclear forces, the form of quantum theory, ... So far, the constraint holds!
- When a new force is proposed, first check whether it satisfies Einstein’s postulates; it needs to have a 4D form.

Conservation of momentum

- Consider a collision between two disks A and B, with the same rest mass, $m_{0.}$ We will look at this collision in 2 frames:
  - The frame of A before the collision.
  - The frame of B before the collision, moving at some v wrt. A.



We pay attention to momentum, p, only along this axis :
Initially, there's no p on that axis.



p_{A2} means momentum of A as seen in frame 2 <u>after</u> the collision.
$p_{A1} = - p_{B2}$ and $p_{A2} = -p_{B1}$ (symmetry)
but $p_{A1} = -p_{B1}$ and $p_{A2} = - p_{B2}$ (conservation)
so $p_{A1} = p_{A2}$ and $p_{B1} = p_{B2}$

How inertial m changes with v.

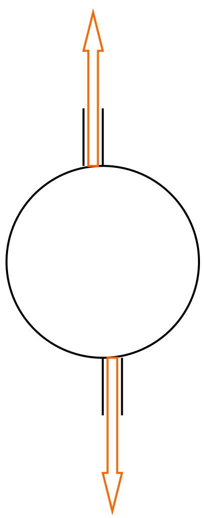
- Now we know that the momentum (along the deflection direction) of disk A is the <u>same</u> in the frame initially at rest with respect to A and the frame initially at rest with respect to B.
- But momentum is mass*distance/time.
- The distances must be the same in both frames. (Why?)
- The elapsed <u>times</u> in the two frames differ by a factor of $\square = 1/(1-\square^2)^{1/2}$, so the mass assigned to disk A in the frame initially moving with respect to A must be $\square$ times as big as the mass assigned to it by frame 1, initially at rest wrt A. So long as we consider the case where the deflection velocity is small, we don't have to distinguish between m in frame 1 and in the frame in which A is now at rest.
- So the frame moving at v wrt A sees A's inertial mass increased over the rest mass by the factor $\square$.

Conservation of Momentum

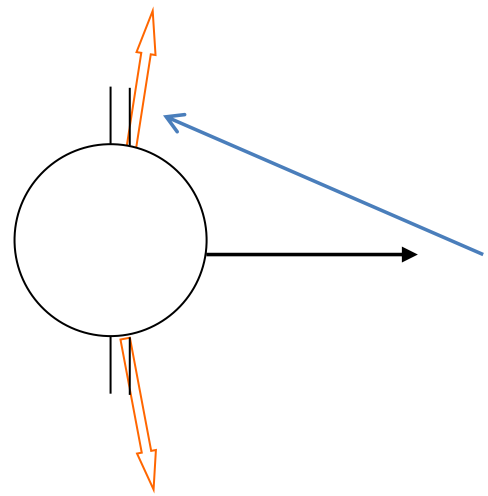
- Assuming conservation of momentum led to the requirement that the m in p=mv is not invariant. $m=\square m_0$
- m_0 is the “rest mass” seen by an observer at rest wrt the object.
- Warning: there's another convention, also in common use, to let "mass" mean what we here call the rest mass, not the inertial mass used above. If you see some apparently contradictory statements in texts, probably that's because they use this other convention.

Energy and momentum

- Consider a star, with two blackened tubes pointing out opposite directions. Light escapes out the tubes, but only if it goes straight out. It carries momentum and energy, known (by Maxwell) to obey $E=pc$.



Now what happens if the star is moving at velocity v to our right?



In order for the light to get out of the moving tubes without hitting the edges, it must be going forward a bit (angle v/c)
So net forward momentum is lost to the light. The lost momentum is: (v/c) □E/c, where □E is the lost energy.

If we assume that total momentum is conserved:
v(□m) = (v/c^2) □E.
So we have a relation between the lost mass □m, and the lost energy □E.

- $\square m = \square E/c^2$
- A reasonable extrapolation is to drop the delta so
- $m = E/c^2$

- This does NOT say that "mass is convertible to energy". It says that inertial mass and energy are two different words for the same thing, measured in units that differ by a factor of c^2.
- This applies to inertial m. *Rest mass* is only a part of that.

Kinetic energy

- So a moving object has energy E = □m_0c^2.
- How does this connect with our usual conception of energy?
  In classical physics, the kinetic energy is
  KE = 1/2 mv^2.
- The time dilation factor : $$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$
- For small β, this is approximately, $$\gamma \approx 1 + \frac{1}{2}\beta^2$$
- Thus: $$E \approx m_0c^2 + \frac{1}{2}m_0v^2$$
- The second term is just Newtonian KE.
  The m_0c^2 term is the energy a massive object has just by existing.
- As long as rest mass is conserved, the m_0c^2 energy is constant and therefore hidden from view. We'll see that rest mass can change, so this energy can be significant.
- Remember that as □ ➔ 1, □ ➔ □. So an object's energy ➔ □. This is a reason why c is the speed limit. It takes an □ amount of energy to get there. As you push on an object, v ➔ c asymptotically. That is, F ≠ ma.

An invariant is lost and another gained

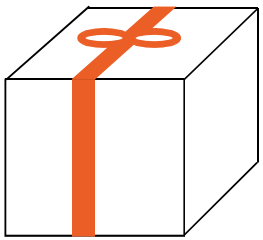
- By assuming the correctness of Maxwell's equations and the principle of relativity we have shown inertial m must depend on the reference frame.
- Lorentz and Poincare' got their speed-dependent m's from essentially the same argument, but using that the laws of physics "look" the same in either frame, not that they are the same.

- So inertial mass is not an invariant.
  - So what's "real" about an object, i.e. not dependent on how you look at it?
- Old invariant: m
- New invariant: $E^2 - p^2c^2 = m_0^2c^4$

Photons (light) have no rest mass

- Newtonian physics does not allow massless objects. They would always have zero energy and momentum, and would be unobservable.
- Now in SR imagine an object with zero invariant mass:
$E^2 = c^2p^2$ so $E=pc$, like for Maxwell's light. Any object with zero invariant mass moves at the speed of light. Gluons are also supposed to be massless.
- Any object moving at the speed of light has *zero* invariant mass, otherwise its energy would be infinite.
- All colors of light (and radio pulses, etc.) from distant objects (e.g. quasars) are found to get to us after the same transit time.

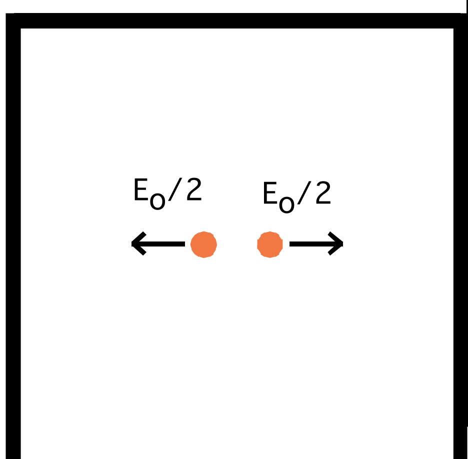
What does “rest mass” mean?



Suppose I have a box with some unknown stuff inside. I want to learn something about what that stuff is by measuring its properties, but I’m not allowed to open the box until my birthday. What can I learn?

I can measure the energy and momentum of the stuff inside by letting the box collide with other objects (assume the box itself to be very light so we can ignore its energy and momentum). Suppose that when the box is at rest (p=0), I measure energy E_o. So the "rest mass" of the stuff is given by $E_o/c^2 = m_0$.

I open the box, only to find two photons bouncing back and forth. Each photon has energy $E = E_o/2$, and since they are moving opposite directions, their momenta cancel ($\mathbf{p} = 0$).



The rest mass of a collection of objects does not equal the sum of their individual rest masses, even if they don’t interact. (unlike inertial mass)
Newton’s concept of mass as “quantity of matter” is gone, although it often remains a good approximation. It’s replaced by a Lorentz invariant relationship between energy and momentum.