What sort of things happen in accelerated frames? Why use them?

Next Topics:

Is curvature necessary? Conventionalism.

Gravitational waves – space is real

Singularities

Global properties of GR – cosmology

Topology, gemoetry

Homework due today!

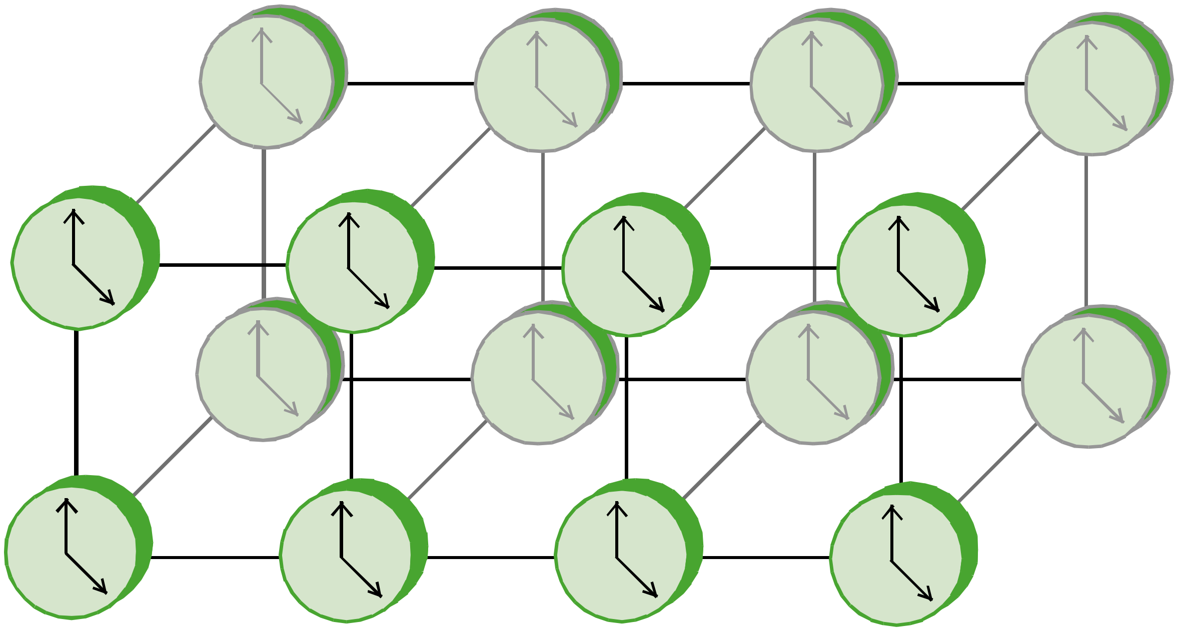
419 students consult about topic

Defining Coordinates

Einstein insisted on the (Machean) idea that each observer must construct his own view of space and time by some actual *observable operations*: "we entirely shun the vague word 'space,' of which, we must honestly acknowledge, we cannot form the slightest conception, and we replace it by 'motion relative to a practically rigid body of reference.' " With no absolute space or time to rely on, we need an operational definition of position and time measurements.

So let's replace Newtonian space & time by an ensemble of meter sticks and clocks

Distribute a set of clocks on a lattice. Clock synchronization is done by sending a round trip signal between two clocks. The signal is assumed to arrive at the second clock halfway between emission and reception by the first. The position of each clock is measured similarly. Half of the round trip time, divided by c, gives the distance.



How do we test if our frame is inertial?

Do Inertial Frames Exist?

- Einstein's operational recipe (make a bunch of identical rods and clocks, build a lattice of the rods to mount the clocks on, check the clock synchrony with light rays) also allows you to *check if Special Relativity is correct*. It *assumes* that once the clocks are synchronized by this procedure, they will stay synchronized. (It also makes other assumptions, but we needn't think about them.)

- So while you are getting used to the ideas that:
  - Aristotle and instinct are wrong, there is no absolute rest frame.
  - Newton and Galileo are wrong, there is no absolute time or distance.

- Einstein has at least opened the possibility that his Special Relativity also is wrong: there may be no inertial frames.

- Einstein's recipe for building a coordinate frame from rigid sticks already has a problem:

Can we follow Einstein's Recipe?

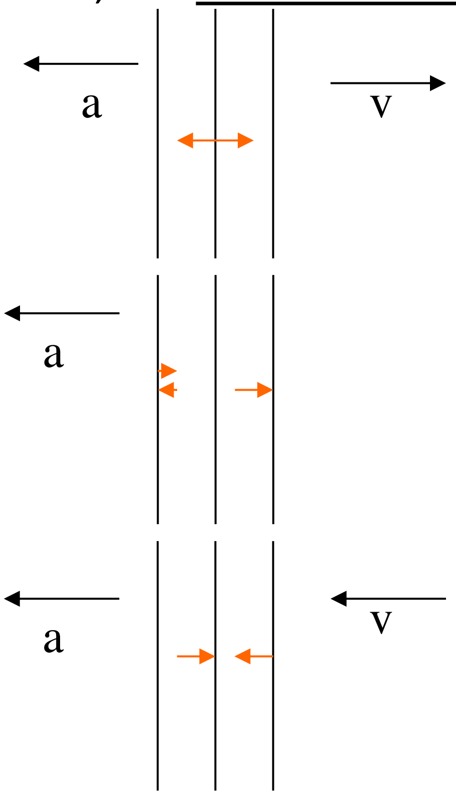
- Rigid bodies cannot exist if Special Relativity is correct.
  - Otherwise you could wiggle one end of a stick and transmit a signal to the other end infinitely fast.

- Each atom only feels forces from its neighbors after a delay- the forces are transmitted at the speed of light. So if a force acts on any part, it always distorts the object at least temporarily.

We also don't know if the clocks will stay synchronized.
So <u>the existence of inertial frames is not simply given</u>.

Accelerating Clocks

- We saw that clock rates must appear different to an accelerated observer. Here's a pair of simple two-mirror clocks viewed over a brief interval during which they accelerate to the left, but <u>at rest in the middle of the time interval</u>. (reference frame)



The light leaves the middle, starting the clock tick on both sides.

The light reaches the left side, which was moving toward it, before reaching the right side, which moved away.

The left-side tick finishes, as the middle side moves toward that light. The right-side tick doesn't, as the middle moves away from it.

<u>An observer can conclude *objectively* that the left clock is running faster than the right clock</u>. The clock the acceleration is *toward* runs *fast*, the clock the acceleration is *from* runs *slow*.

Time and Position

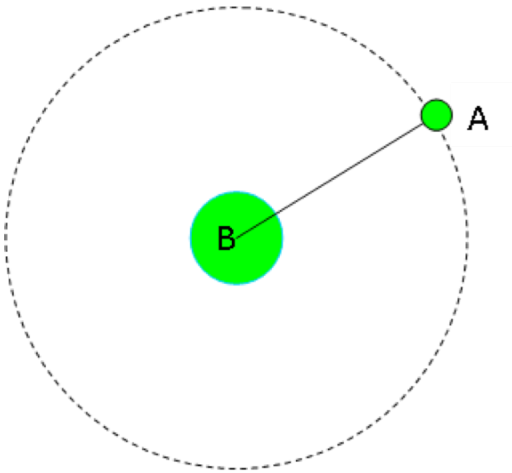
- We could make a whole stack of these clocks, applying the same argument to each. The farther the clocks are away in the direction the acceleration is toward, the faster they run. The farther they are away in the opposite direction, the slower they run.

- Everybody agrees on this, even if they can't agree on which clock is "right". (*unlike* disagreement between inertial frames, in which they can't agree on who's faster.)

- Notice that this effect is just what we needed in the twin "paradox".
As Alice accelerated <u>toward</u> Beth, (while turning back) she concluded that Beth's clock's were running fast.

How Big an Effect?

Let's calculate the effect of the acceleration is on clock rates:



B is a massive object,
A is less massive, held in orbit by a string.
B ~doesn't accelerate.
B says A's clock runs slow by a factor of:

$$\frac{1}{\gamma} \equiv \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

What does A say about B?

If A weren't accelerating, she would see B as slowed by a factor of 1/γ. But since A circles B repeatedly, they can compare times on each orbit, with <u>no change in signal transmission times</u>. So A and B must agree about who is faster and by what factor. Thus A also thinks that B's time runs faster by a factor of γ . The net effect of A's acceleration in speeding the rate at which A sees B's clocks run is thus a factor of:

$$\gamma^2 = \frac{1}{1 - \frac{v^2}{c^2}} = \frac{1}{1 - \frac{\vec{a} \bullet \vec{R}}{c^2}} \approx 1 + \frac{\vec{a} \bullet \vec{R}}{c^2}$$

This agrees with our argument from stacked light clocks: the clock rate increases with the distance in the direction of the acceleration, decreases the other way. This is also just the amount needed to get Beth and Alice to agree. (We won't try to make the argument accurate beyond the linear term in the acceleration- we would have to worry about who measures "a" and "R".)

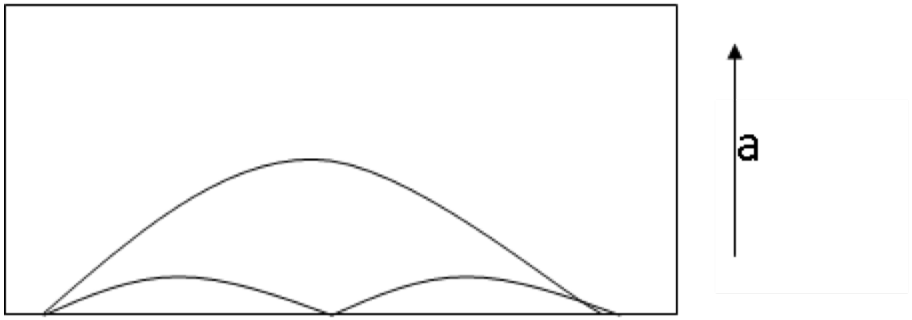
The Accelerating Observer

sees something very strange. In her reference frame, identically constructed clocks run at different rates, depending on where they're located!

- It would seem that a reasonable law of physics should be that identically constructed clocks run at the same rate. This sort of gross effect should tell A that she is accelerated.
- In an accelerated frame, you cannot go through the exercise of building a lattice of identical synchronized clocks to define the coordinate system: the clocks don't stay synchronized.
- It certainly looks like accelerated frames are a curiosity, since in one we would have to abandon some simple laws of physics.
- At one historical point, the only reason to insist on looking at the laws that apply within accelerated frames was Einstein's Machian prejudice that the laws of physics should depend only on the relations among objects, not on absolute motion in *any* sense.

Oddities of Accelerating Frames

Other strange things happen in uniformly accelerated frames. A light ray travelling at right angles to the acceleration seems to bend, as if it were falling in the direction opposite to the acceleration.

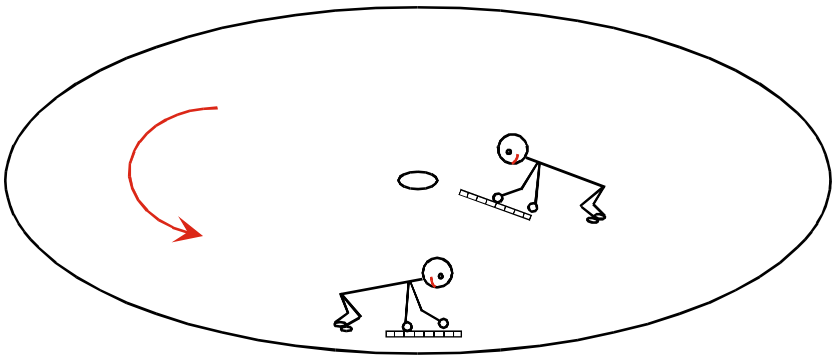


Straight lines are supposed to be the shortest paths between two points. Now the quicker path here *is* the path of the uninterrupted light ray (we can see that easily in the lab frame) but in the accelerated frame the spatial path length of that light ray seems *longer* than the path of the ray that makes a bounce off the wall. Isn't the speed of light supposed to be constant? If we tried to make our ordinary laws of physics work in such a frame, we couldn't identify light rays' paths with straight lines.

How would we define straight lines?

Non-uniform Acceleration

As long as we're looking at weird reference frames, let's see what would happen if you used a *non-uniformly* accelerating reference frame, e.g. a merry-go round (MGR).

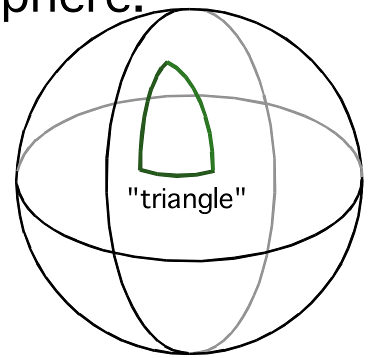


Before the MGR accelerates, you get a bunch of little meter sticks. If you measure the circumference C and radius R by counting out meter sticks, you find $C = 2\pi R$, whether you do this on the MGR or the ground. Now set the MGR spinning. It will stretch, etc., but you tighten down any bolts needed to make its circumference still fall exactly above the previous circumference, traced out on the ground. If you measure on the ground, you get the same old C and R. Due to the Lorentz contraction, the rulers measuring the circumference on the MGR have shrunk, but not the ones used for the radius. Therefore, in the MGR measurement, $C > 2\pi R$. Not only that, the ratio C/R depends on the radius of the circle. (It gets bigger for bigger R.)   This is not Euclid's plane geometry, but rather resembles the sorts of geometries you get if you try to confine measurements to curved surfaces. No wonder it's hard to find straight lines with familiar properties!

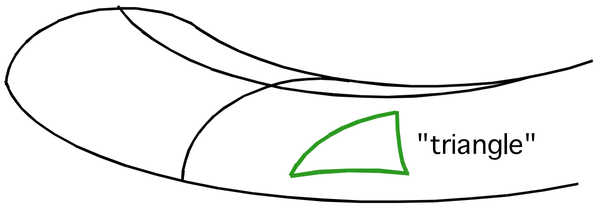
Curved Geometries

Here are 2-d surfaces in 3-d in which $C \neq 2\pi R$

Sphere:



Saddle:



If we want to accept non-uniformly accelerated frames as equally valid, we must accept such weird geometry as being the correct description of our 3-d space. In other words, we should be able to distinguish whether we are using a non-uniformly accelerated frame by whether measurements confirm Euclid's axioms.

Our Choice of Frames

We already saw that the outside observers in a rotating frame think that the clocks at the middle are running fast. So if we try to use a rotating frame, clock rates depend on position.

We have a choice:

1. Reject accelerating reference frames, because they

require clock rates to depend on position,

violate Euclidean geometry (with reasonable definitions of length),

generally make a mess out of familiar laws of nature.

2. Accept accelerating frames, and make new laws of nature that have all those weird effects

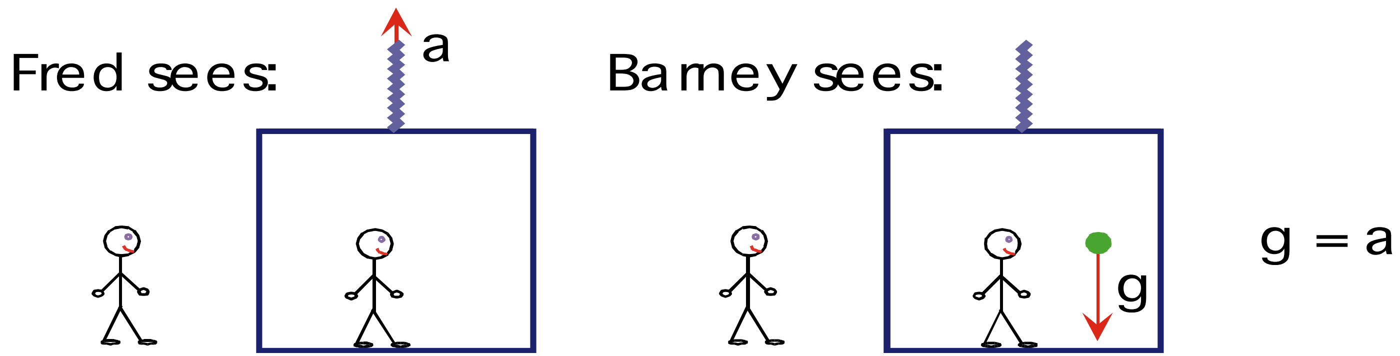
- **Anyone in his or her right mind would choose (1).**
- **Why then do physicists choose (2)?**

The equivalence principle

- “The gravitational mass of a body is equal to its inertial mass.”
- Remember: inertial mass is the m that appears in $p = mv$ or $E = mc^2$, or (approximately) $F= ma$.
- Gravitational mass is the m that appears in Newton’s law of gravity: $F = GMm/r^2$. It tells us the strength of the gravitational force between two masses. It has been empirically determined that the two kinds of mass are exactly the same to very high accuracy(10^{-17}). In other words, different types of mass all show the same acceleration (from a given starting velocity) in a gravitational field independent of what they are made of.
- When you calculate the gravitational acceleration of an object using $a = F/m$ the object's own mass, *m drops out* so that $a = GM/ r^2$.
- The result is that gravity makes every object accelerate together: the effect of gravity is completely describable classically by an acceleration field, as has been known since Galileo’s time.
- That means that you don't feel gravity in the same way that you feel other forces. Since all your parts are accelerating together, gravity creates no strains, tickles no nerves…
  - However, as Einstein put it, he was the first to “interpret” this fact.

The equivalence principle

- Consider the "elevator" gedanken experiment. We are somewhere in intergalactic space, with no planets or other junk nearby. Fred is resting at ease in his un-accelerated reference frame. Barney, on the other hand is inside a box and can't see out.
- Suppose there is a rope attached to the box, and some external agent pulls on the rope, accelerating the box at exactly 9.8 m/s^2.



Fred says: "The box (and Barney) are accelerating. So what?"
Barney says, "I am not accelerating. I am in an elevator which is hanging from its cable in a gravitational field. It's the same field that's making Fred fall, because no cable supports him." Who is correct?

Einstein insists that in the absence of a reason for preferring one point of view, one must accept both.

Einstein proposed another generalization

- No measurement of any sort can detect a uniform gravitational field.

And, by the way:

- No *local* measurement can detect *any* gravitational field.

If we accept Einstein's generalization…

We cannot distinguish an accelerated frame from a gravitational field. We said that no sane person would voluntarily accept accelerated frames, because they lead to all sorts of crazy effects.

- If gravity were completely uniform, you could get rid of it by transforming to another reference frame. Gravity can always be eliminated that way in a small region (*e.g.*, inside Barney's elevator), but not over a large spacetime domain, because there is an uneven distribution of matter.

- So no sane person can reject a universe with gravity: you can't get rid of gravity without getting rid of everything. The gravity isn't uniform, so our actual reference frames are like the ones with non-uniform accelerations.

- If Einstein is right, then a world with gravity has all those bizarre effects we found for accelerating frames, whether you like them or not.