

Physics 419
Lecture 12: Accelerated Frames
March 4, 2021

1 Themes

- How do you feel gravity?
- Einstein's Equivalence Principles
- Acceleration is Curvature

2 Special Relativity Redux

Aside from its profound experimental consequences (length contractions and time dilation), special relativity is important because it establishes the fundamental invariance principle that ties together the Newtonian world and that of Maxwell. The key principle is that uniform motion does not change the laws of physics and hence must leave both theories unchanged.

An important consequence of the principle of relativity is that if the laws of physics are to be the same in every inertial reference frame, the quantities on both sides of the = sign must undergo the same Lorentz transformation.

Consider momentum. In Newtonian physics, $\mathbf{p} = m\mathbf{v}$ (bold means vector). Momentum and velocity are vectors, and mass is a scalar (invariant) under 3-d rotations. This equation is valid even when we rotate our coordinates, because both sides of the equation are vectors.

We want to generalize this to 4-d. The natural (and correct) answer is that momentum is a 4-d vector (4-vector for short). That is, in addition to the x, y, and z components that it used to have, momentum now has a "t" component as well. The fourth component of momentum is E/c , the energy. The factor of c is needed to give it the same units as momentum.

Just as the lengths of 3-vectors remain unchanged under rotations, there is an invariant "length" of 4-vectors under Lorentz transformations. The length of a 4-vector is the square of its time component minus the square of its space component: $(E/c)^2 - p^2 = (m_0c^2)^2$

Notice that you cannot make any invariant from space or time variables alone. That's why we call the SR world is 4-D, and call the old world 3-D + time. No true feature of the world itself is representable in the 3 spatial dimensions or the 1-time dimension separately.

The Replacement of Newtonian Space and Time by an Ensemble of Meter Sticks and Clocks:

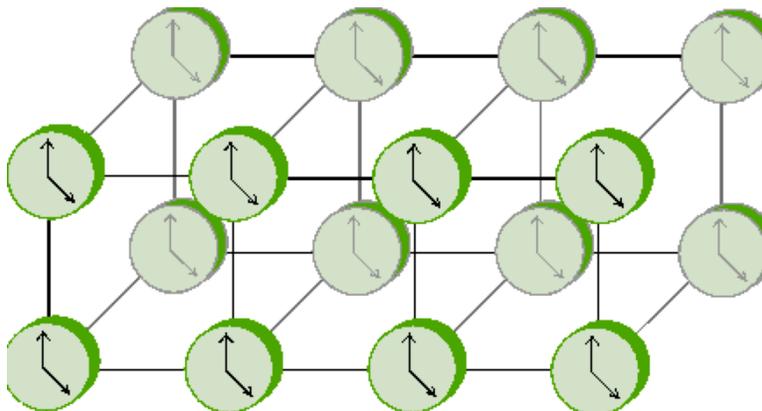


Figure 1: Clocks and meter sticks to demarcate Einsteinian spacetime in special relativity.

The establishment of a coordinate system for measuring position and time is done by distributing a set of clocks on a lattice. Clock synchronization is done by sending a round trip signal between two clocks. The signal is assumed to arrive at the second clock halfway between emission and reception by the first.

The position of each clock is measured similarly. Half of the round trip time, divided by c , gives the distance.

Why so complicated a specification? There is no absolute space or time to rely on, so we need an alternative. Without an operational definition of position and time measurements, it would be difficult to understand the odd effects that we'll encounter.

Einstein's operational recipe (make a bunch of identical rods and clocks, build a lattice of the rods to mount the clocks on, check the clock synchrony with light rays) also allows you to check if relativity is correct. It assumes that once the clocks are synchronized by this procedure, they will stay synchronized. (It also makes other assumptions, but we needn't think about them.)

Redux

So it seems that Newton and Galileo are wrong— there is no absolute time or distance.

Einstein has at least opened the possibility that his Special Relativity also is wrong; there may be no inertial frames.

3 GR Precursors

Special relativity applies to inertial reference frames, that is frames that are not accelerating. Hence, it does not apply to frames in which gravity is acting. Einstein hunted for the general theory that should apply when gravity was present. The first problem Einstein had to solve was gravity, namely how do we really know that we are being acted on by gravity?

- Where do you feel gravity?
- Do you feel gravity when you're falling?
- Does gravity change when you land?
- If you have a collection of particles in a uniform gravitational field, how are their mutual distances, velocities, etc affected by the field?

Newton argued that acceleration was absolute with his famous rotating pale of water example. The fact that the surface of water was concave proved to Newton that the effects of acceleration were absolute. Mach generalized this by considering a Gedanken experiment in which two pales of water were placed along the same axis and one was rotated relative to the other. If these are the only two objects in the universe then it would be impossible to tell which was rotating relative to the other. Hence, the surface of both must look the same. Hence, acceleration is not absolute. He argued that the concavity of the water in the single pale case is caused by the mass of the distant galaxies, etc. As a result acceleration is relative to the distant stars, etc.

Einstein was heavily influenced by this Mach principle and started his path to a theory of general relativity from it. General relativity is based on two equivalence principles:

- 1.) The gravitational mass of a body is equal to its inertial mass.

Inertial mass is the m that appears in $\mathbf{p} = m\mathbf{v}$ or $E = mc^2$, or (approximately) $\mathbf{F} = m\mathbf{a}$. It tells us how much inertia (resistance to being accelerated) an object has.

Gravitational mass is the m that appears in Newton's law of gravity: $F = GMm/r^2$. It tells us the strength of the gravitational force between two masses. It has been empirically determined that the two kinds of mass are exactly the same to within a part per trillion (10^{-12}).

Notice that when you calculate the acceleration of an object using $a = F/m$ the object's own mass, m

DROPS OUT,

$$a = GM/r^2. \tag{1}$$

The result is that gravity makes every different type of object accelerate together: the effect of gravity is completely describable classically by an acceleration field, as has been known since Galileo's time.

That means that you don't feel gravity in the same way that you feel other forces. Since all your parts are accelerating together, gravity creates no strains, tickles no nerves.

However, as Einstein put it, he was the first to "interpret" this fact. The bending of light by gravity follows strictly from this equivalence principle.

2.) No local measurement can detect a uniform gravitational field.

Consider the famous "elevator" gedanken experiment (E called it a chest). We are somewhere in intergalactic space, with no planets or other junk nearby. Fred is resting at ease in his unaccelerated reference frame. Barney, on the other hand is inside a box and cannot see out.

Suppose there is a rope attached to the box, and some external agent pulls on the rope, accelerating the box at exactly 9.8 m/s^2 .

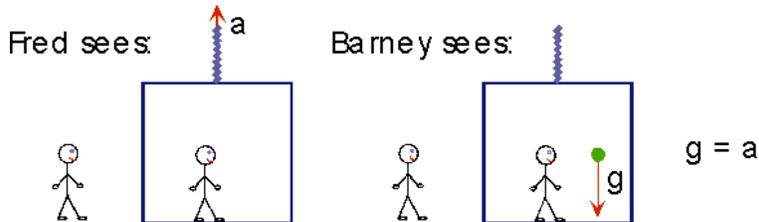


Figure 2: Elevator gedanken experiment of Einstein's.

Fred says: "The box (and Barney) are accelerating. So what?"

Barney says, "I am not accelerating. I am in an elevator which is hanging from its cable in a gravitational field. It's the same field that's making Fred fall, because no cable supports him."

Who is correct? Einstein insists that in the absence of a reason for preferring one point of view, one must accept both. This is the strong principle of equivalence, namely that all phenomena are equally describable by either picture. The weak principle of equivalence asserts that the gravitational and inertial masses are the same.

If we accept Einstein's generalization we cannot distinguish an accelerated frame from a gravitational field. We said that no sane person would voluntarily accept accelerated frames, because they lead to all sorts

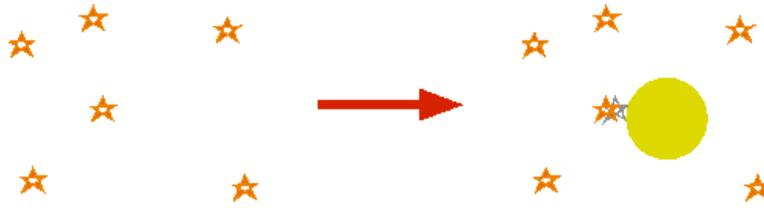


Figure 3: Light rays being bent by the sun.

of crazy effects.

If gravity were completely uniform, you could get rid of it by transforming to another reference frame. Gravity can always be eliminated in a small region (e.g., inside Barney's elevator), but not over a large spacetime domain, because there is an uneven distribution of matter.

So no sane person can reject a universe with gravity: you can't get rid of gravity without getting rid of everything. So we can replace gravity by any accelerated frame. This is key and a great simplification in thinking about gravity—just think about acceleration.

An unexpected consequence of the equivalence principle is that light rays bend in a gravitational field. Let's just use $E = mc^2$. The equivalence of mass and energy says that light rays have a mass. Based on the equivalence principle this mass must act like a gravitational mass. Hence, a light ray should fall toward the gravitational source. Note, this means that light is accelerated in a gravitational field. Hence, its velocity is not constant!!!! Waw, this is weird. Didn't we spend alot of time extolling the genius of Enistein and marveling at Michelson and Morley who collectively taught us that the speed of light is constant? It is this apparent contradiction that led to Einstein's theory of general relativity. This theory is much more subtle than special relativity and took much longer to construct. We will see that the answer lies in what we mean by falling in a straight line. General relativity predicts that light rays from a distant star grazing the surface of the sun should be bent by 1.75 seconds of arc. This result is twice that predicted by just using $m = E/c^2$ and then gravity. Nonetheless it was verified by a total eclipse in 1919. During an eclipse, the light from a star that is just grazing the sun can be seen quite clearly. This should provide a displacement of the star. This displacement was verified in 1919 and heralded the dawn of the Einsteinian world and the demise of Newton. Things would never quite be the same after that. Basically, nothing was left of Newton. Quantum mechanics was just being formulated and that would signal the end of the purely classical interpretation of what there is. CONFIRMED, 1919 solar eclipse

4 Curved spacetime

Straight lines are supposed to be the shortest paths between two points. Consider living on a basketball. What is the shortest distance between the two points on the equator but on opposite sides? One cannot burrow through the basketball. Hence, the shortest distance is not the diameter but half the circumference, namely πR . Note this is greater than the diameter, $2R$. To you, you are moving as straight as you can. Hence, there is nothing odd that you detect. The same thing is true of light if the effects of gravity are to produce curvature in the universe. Since we cannot distinguish between an accelerated frame and one being acted on by gravity, then all we need to do is figure out what happens in the presence of some arbitrary acceleration. We go now to the famous merry-go-round example.

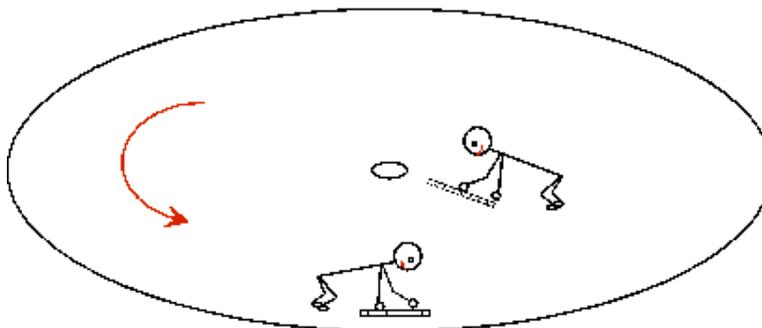


Figure 4: Measuring a merry-go-round with measuring sticks.

Before the merry go-round (MGR) accelerates, you get a bunch of little meter sticks, all stacked together and the same length. If you measure the MGR circumference C and radius R by counting out meter sticks, you find $C = 2\pi R$, whether you do this on the MGR or the ground. Now set the MGR spinning. It will stretch, etc, but you tighten down any bolts needed to make its circumference still fall exactly above the previous circumference, traced out on the ground. If you measure on the ground, you get the same old C and R .

Due to the Lorentz contraction, the rulers measuring the circumference on the MGR have shrunk, but not the ones used for the radius. Therefore, in the MGR measurement, $C > 2\pi R$. Not only that, the ratio C/R depends on the radius of the circle. (It gets bigger for bigger R .) This is not Euclid's plane geometry, but rather resembles the sorts of geometries you get if you try to confine measurements to curved surfaces. No wonder it's hard to find straight lines with familiar properties!

Geometries in which $C/2R \neq \pi$.

Here are 2-d surfaces in 3-d that have each property.

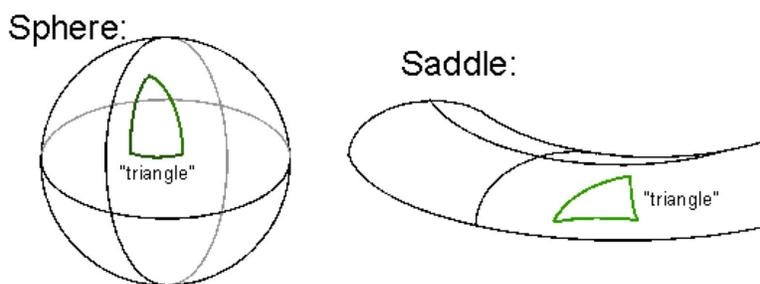
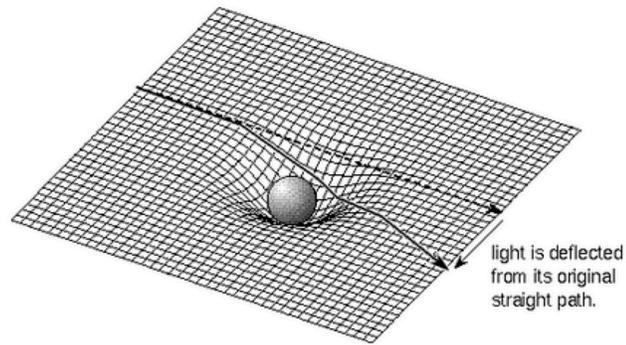
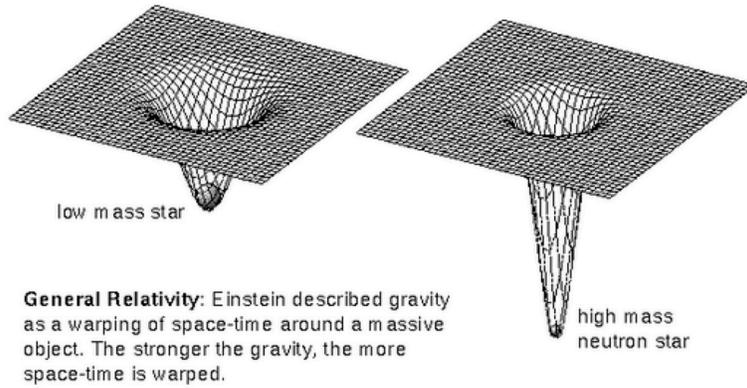


Figure 5: Sphere and a saddle geometries and what a triangle would look like on them.

On a sphere, the diameter is $D = \pi R$ but the circumference is still $C = 2\pi R$. Consequently, $C/D < \pi$. On a saddle, $C/D > \pi$ as in the merry-go-around case. Hence, the geometry implied by acceleration produces concavity not convexity. This is illustrated in the last figure.

Don't worry about whether the space looks like a piece of some hypothetical Euclidean higher dimensional space, which would have absolutely no physical significance.



General Relativity: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent.

Figure 6: Curvature of spacetime in General Relativity.