Lecture 16/17: 18 March , 2021

PHYSICS 419 - Spring 2021

I. Introduction

In this lecture I am going to introduce the main conceptual features of quantum mechanics by focusing on the two-slit experiment done several different ways. Most texts and teachers of quantum mechanics bury the physics in needless mathematizing. I will avoid this as much as possible. Today we will start extracting from two experiments the kernel of quantum mechanics: a) uncertainty necessarily implies quantum interference, b) quantum interference necessarily implies uncertainty, and c) quantization of measurement.

II. Uncertainty and Quantum Interference

a.) Two-slits and marbles

Consider a marble dispensing device that sprays marbles at a screen into which two slits are carved. Behind the screen is a wall with a detector. The detector is movable and traps the marbles that hit the wall. The experimental set-up is shown in Fig. (1). Given that a marble can bounce off the screen or deflect off one of the slits (or both) we ask the question, what is the probability that a marble strikes the detector a distance x from the center as indicated above. Lets assume that the marbles do not fall apart when they strike the detector. We say then that marbles come in identical lumps, to quote Feynman. The lumpiness of marbles is unaffected by the rate at which they are fired from the dispenser. Let us first cover up slit 2 and measure the probability that marbles pass through slit 1. We expect a bell-shaped curve of the form shown in Fig. (1). An identical curve is expected for P_2 measured in a similar manner. Now lets open both slits and measure P_{12}. We find that the result is just the sum of P_1 and P_2. Hence, there is a peak in P_{12} at $x = 0$, the center. This is all that happens with marbles. In situations in which $P_{12} = P_1 + P_2$, we say that there is no interference. Marbles arrive at the wall in non-interfering lumps.

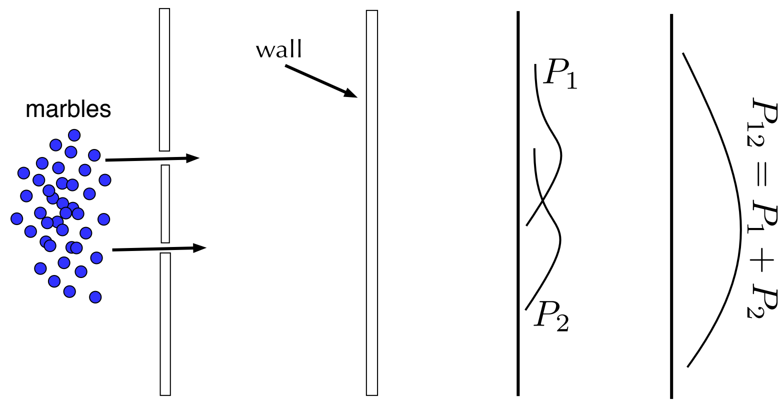

Figure 1: Marbles impinging on a screen. The probability that a marble is detected at a particular height is plotted when slit 2 is closed and slit 1 is open. This is given by P_1. The equivalent probability for slit 2 open and slit 1 closed is P_2. The joint distribution is P_{12} which is observed to be the sum of the two.

b.) Two-slits and waves

Let us repeat the same thing with a wave generating device. The experimental set-up is shown below in Fig. (2). An essential difference between marbles and waves is that waves do not come in identical lumps. What the detector measures is the intensity or height of the wave. The height can vary drastically depending on which part of the wave hits the detector. It makes most sense to ask then, what is the intensity of a wave a distance x from the center. Let us first perform the experiment with slit 2 covered. As in the marble problem, we obtain an intensity curve that is bell-shaped centered around the position of slit 1. The same thing is observed for the intensity through 2 with 1 covered. When both slits are open, however, waves entering through slits 1 and 2 can interfere. As a result $I_{12} \neq I_1 + I_2$. The correct intensity curve is the last shown in Fig. (2). In general a wave can be specified as $he^{-i\omega t}$ where h, is a complex variable that depends on the position, with ω the frequency of the

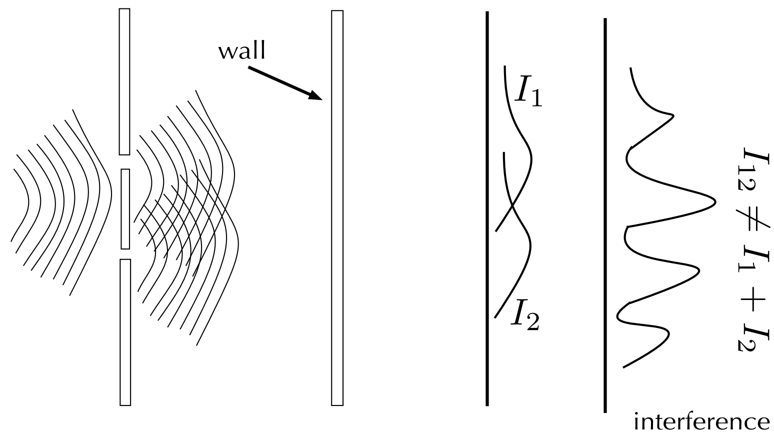

Figure 2: Waves impinging on a screen. Here we plot the intensity of he wave at a particular height. P_1 and P_2 have have the meanings as before. We see in this case that as a result of interference, the joint distribution is not a sum of the two.

wave. For waves entering slits 1 and 2, the intensity measured at the detector is given by

$$I_{12} = |h_1 + h_2|^2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2|cos\delta, \tag{1}$$

where δ is the phase difference between the two waves. The last term in Eq. 1 describes interference. The minimum value is obtained when the phase difference between the waves is π. At such points the overall intensity is the modulus squared of the difference between the two waves entering 1 and 2. In general, destructive interference is expected whenever the distance between hole 1 and the detector is different from the distance between hole 2 and the detector by an odd number of half-wavelengths. The minima in I_{12} correspond to these cases. The maxima indicate constructive interference between the waves entering slits 1 and 2. It is the inherent non-lumpiness of waves that gives rise to interference. If waves were lumpy, the interference would go away.

c.) Two-slits and electrons

Let us now do the same experiment with electrons. Of course, the experiment is somewhat

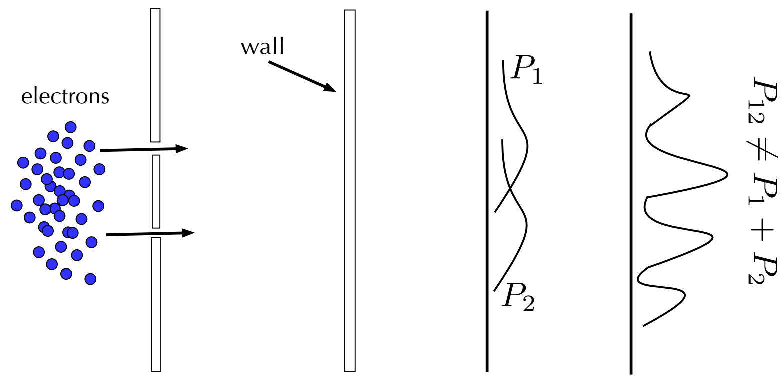

Figure 3: Electrons impinging on a screen. P_1 and P_2 have the same meanings as before. We see in this case that as a result of interference, the joint distribution is not a sum of the two. Nonetheless, electrons do come in discrete units and hence are like marbles. What is going on here?

hypothetical. Or is it. This experiment (see https://www.nature.com/articles/44348.pdf) was actually done with buckyballs (a 60-atom carbon structure) and the results reported here were observed. Since buckyballs are much bigger than electrons, if interference is observed here, there is no doubt that the same will occur for electrons. Let us now envision that the source of electrons is a heated tungsten wire and the detector is a Geiger counter. The experimental set-up is shown in Fig. (3) The detector makes a click each time an electron strikes it. One click implies one electron. A fraction of an electron can never arrive at the detector. Hence, electrons are lumpy. This is a crucial point. Let us now measure the joint probability P_{12}. Because electrons are lumpy as far as our detector is concerned, we would expect P_{12} to look like it did in the case of marbles. However, if we were to perform this experiment, we would observe quite a different result. The actual result is shown above. This is identical to the water wave result. We have established however, that electrons are lumpy. That is, they are particulate. We cannot account for this behaviour by invoking complicated paths along which the electrons travel. From where then does the observed interference come? Let us try to construct a theory of the joint probability curve shown above. We know we have to end up with quantum mechanics, so this is a good check on our speculations.

Let us begin by testing the hypothesis: Each electron either goes through hole 1 or it goes through hole 2. If this statement were true, then we can divide the electrons into two groups: those that went through hole 1 and those finding hole 2. The test of this hypothesis is simple then. We have to measure P_1 and P_2 independently. If the hypothesis is true, then it must be that $P_{12} = P_1 + P_2$. We 'verify' this hypothesis by sticking our eyeball at hole 1 or 2 and looking at the electrons whizzing by. To facilitate our observations, we shine a light behind the screen. The light scattered by the electron will be deflected to our eye and we will be able to 'see' the electron. This is what we will see. When then hear a click and we will then record which hole the electron went through. We will then form a histogram as a function of frequency of hits and position. Experimentally, we are bound to find a distribution identical to the marble case for P_1 and P_2. Hence, the joint distribution from such an experiment is the smooth bell curve centered at zero as shown in Fig. (4).

This is of course not what is seen (see Fig. (3)) when both slits are open. What is wrong with our experiment? Let us now repeat the experiment when the light is turned off. We find that the distribution showing the interference is restored. This means that the light is the culprit. When an electron is scattered by light, its motion is altered markedly. The exact effect can be worked out knowing the form of the electric field of the light and that an electric field exerts a force on a charged particle. Hence, by measuring P_{12} in the presence of photons, we actually are deflecting the electrons in such a way that the interference pattern is seriously altered.

Can we minimize the effects of the light? We can try dimming the light. By dimming

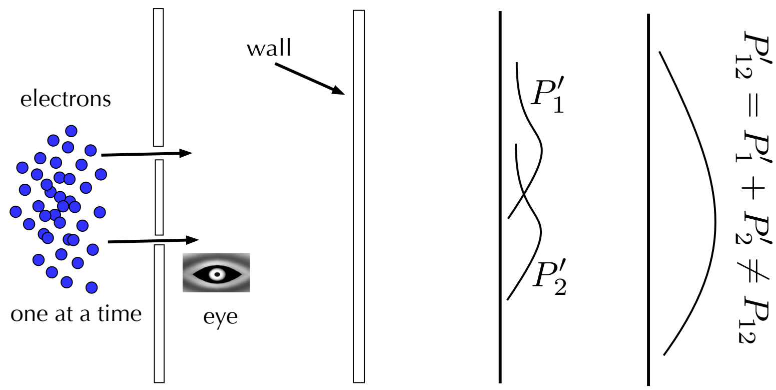

Figure 4: Electrons impinging on a screen but with a measuring device indicated by 'eye' to determine which slit the electrons came trough. Lets for the sake of argument send the electrons in one at a time. P'_1 and P'_2 have the same meanings as before. We see in this case that as a result of interference, the joint distribution is the sum of the two but not equal to the original interference pattern.

the light, we are changing the rate at which photons are being emitted per unit time. Hence, when an electron goes whizzing by, there may not be a photon around to scatter the electron. In our experiment then, we find that sometimes we hear a click without a flash of light. In these instances, we cannot tell which slit the electron went through because the electron has gone by without being seen. We then keep track of all the electrons that are not seen by recording the position of the detector when a click is heard. We find that this distribution does in fact follow P_{12} of the wave interference case. Because the only electrons that are seen are the ones that are scattered by the light and these electrons obey the lumpy joint distribution, we conclude that interference cannot be seen. If it cannot be seen, can we falsify our hypothesis?

There is one last modification we can make of our experiment. We can try increasing the wavelength of the light. Here I will appeal to an observation made by Louis de Broglie. In trying to grapple with quantum phenomena, he reasoned that all particles have wavelengths. We can introduce the de Broglie formula by focusing on what happens when we increase the wavelength of a wave. The peaks are now more spread out. This should decrease the energy and hence the momentum should go down as well. This is what de Broglie proposed. His formula is that the momentum, p, changes with wavelength as $p = h/\lambda$ where h is Planck's constant and λ the wavelength. Its hard to believe that such a simple idea was awarded the Nobel Prize, once again showing that many times less is more. Hence, we can slow the photons down by increasing their wavelength. We reason that if the photon momentum is sufficiently weak then the deflection that is caused by the light will not be too great and we will be able to see which slit the electron goes through. The problem, however, is that as the wavelength of light becomes comparable to the distance between the slits, the scattered light becomes a big fuzzy blur completely obscuring the precise hole into which the electron went. This is the uncertainty principle at work. It gets in the way of knowing which hole the electron entered. Heisenberg noticed this and proposed as a general principle, "It is impossible to design an apparatus to determine which hole the electron passes through, that will not at the same time disturb the electrons enough to not destroy the interference pattern". This is the actual statement of the uncertainty principle as stated by Heisenberg. Stated in this way, it lays plain that quantum phenomena hinges precariously on the limitations of an experimental apparatus. What about our original hypothesis? We are bound by the uncertainty principle never to be able to falsify it. We cannot tell which slit the electrons went through because we would be necessarily destroying the quantum interference which created the joint distribution function to start with. Uncertainty and quantum interference go hand in hand and are at the heart of quanutm phenomena. The only way of describing the joint distribution observed for electrons is to invoke that an electron entering through slit 1 is described by a complex number ϕ_1 and those entering slit 2 by ϕ_2. The probability of observing an electron at the detector a distance x from the center is $P_{12} = |\phi_1 + \phi_2|^2$. We will refer to ϕ as the state vector or the probability amplitude of the electron. Here are the general principles that the two-slit experiment has taught us.

a) The probability of an event in an ideal experiment is given by the square of the complex number ϕ: $P = |\phi|^2$.

b) When an event occurs in several alternate ways, the probability amplitude for the event is the sum of the individual probability amplitudes. (This is the general statement of the superposition principle). The square of this quantity defines the probability for the event. The cross terms give rise to interference. For the two slit experiment we have that

$$\phi = \phi_1 + \phi_2 \tag{2}$$

$$P = |\phi_1 + \phi_2|^2 \tag{3}$$

c) If an experiment is performed which is capable of determining whether one of two alternatives obtains, then the joint probability is simply a sum of the independent probabilities for each alternative. That is, interference is lost and the classical picture applies.

$$P = P_1 + P_2 \tag{4}$$

These three statements encapsulate much of the conceptual nature of quantum mechanics. Of course, there is much in these statements and a host of profound physical results follow as well as mathematical theorems. One thing should be said here. The hypothesis we made regarding the two slits is useless because it cannot be tested. That is, it cannot be falsified. If it could, we would only be observing classical phenomena. Within quantum phenomena, we are chained to uncertainty. We can only talk about the probability that an event will occur.

III. Size of an Atom

If the uncertainty principle is accurate, it should tell us the minimum size of the smallest atom. The usual way the uncertainty principle is stated is that if the position along an axis is measured within an accuracy of Δx, then the uncertainty in the momentum must be at least $\Delta p = h/\Delta x$. An electron in an atom cannot spiral inwards, else its position would be known precisely. It has some average distance it maintains from the nucleus. At any time we can specify the probability amplitude ϕ which tells us the amplitude that the electron is at a given position and time. Let us call the average electron orbital distance a. The spread in the momentum scales as $\frac{h}{a}$. Lets assume that this is also the maximum momentum. This is equivalent to assuming that a is the distance of closest approach to the nucleus. Let us now estimate the kinetic energy of the electron: $K.E. = \frac{p^2}{2m} = \frac{h^2}{2ma^2}$. We see then that the kinetic energy scales as h^2. This is important. The only thing we have to add in to estimate the total energy is the Coulomb energy between the electron and the nucleus. This quantity scales as $-e^2/a$. The total energy is then $E = \frac{h^2}{2ma^2} + \frac{-e^2}{a}$. Lets now minimize with respect to a. This will give us the minimum size of the electron orbit. We obtain,

$$dE/da = -\frac{h^2}{ma^3} + \frac{e^2}{a^2} \tag{5}$$

Solving for a, we find that $a_{min} = h^2/me^2 = .528\text{Å}$. This is the Bohr radius. If we substitute this value back into our energy equation, we find that the minimum energy is $E_{min} = \frac{-e^2}{2a_{min}} = -13.6eV$. This is the binding energy of a 1s electron in a hydrogen atom. It is these sorts of results that quantum mechanics should tell us. We obtained this result by minimum math–no need to solve the Schrödinger equation, which we will not do in this class.