PHYSICS 419 - Spring 2021

Lecture 25: April 22, 2021

1 AMPS Paradox

Around 2013, Almheiri, Marolf, Polchinski, and Sully (AMPS, J. High Energy Physics (2013) 2013:62) discovered a curious feature of entanglement near black holes which required a bizarre solution. This lecture is about that apparent paradox and the current thinking about it. To put the problem in context, we need to review monogamy of entanglement. Two particles that are entangled cannot be described by a separable wavefunction–that is a matrix product state. Consider some property of the particles that is described by observables $\otimes$ and $\hat{\otimes}$. Maximally entangled pairs always come in the EPR form of

$$|\mathrm{MAX}\rangle = \frac{|\otimes_1, \hat{\otimes}_2\rangle \pm |\hat{\otimes}_1, \otimes_2\rangle}{\sqrt{2}}. \tag{1}$$

Although this state represents everything there is to know about the two particles quantum mechanically, it is somewhat disappointing because it does not tell us precisely which state the particles are in vis a vis $\otimes$ or $\hat{\otimes}$. This is quantum entanglement. The description of the whole system says nothing about the constituents. If the particles were sent to detectors infinitely separated as in the EPR thought experiment, any measurement on either particle completely specifies the state of the other although no direct measurement of the other particle is made. This is maximal entanglement. Monogamy of entanglement states that any two EPR pairs can only share that relationship with themselves and no other particles. This seems physically sensible because the basis $\otimes, \hat{\otimes}$ is complete. We can create a macroscopically entangled collection of such particles by simply sending them to boxes separated by a large distance as shown in Fig. (**??**).

As we mentioned last class, the vacuum in quantum mechanics is not empty. It consists of a teeming collection of virtual particle-antiparticle pairs that have no net energy. Let's divide space down the middle and construct the possible states. Assume what we are measuring are virtual particle-anitparticle pairs. Across our partition, the cell immediately to the left or right either has a particle or does not. We can represent this with a state of the form

$$|\mathrm{Space}\rangle = |\circ, \circ\rangle + |1, 1\rangle. \tag{2}$$

Note, this state represents all possibilities and hence is a maximally entangled state of the kind in Eq. (**??**). Hence, across the partition, the correct description of the particle states is one of entanglement as

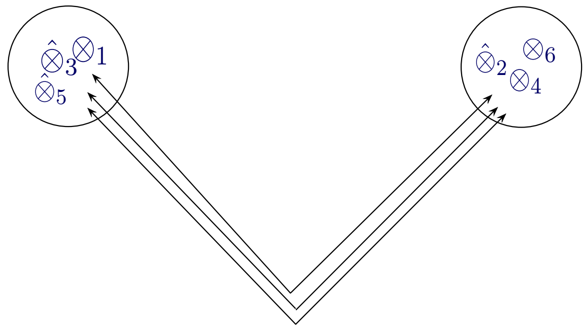

Figure 1: Separation of entangled pairs into two distinct bins. Each particle knows which particle in the other bin is its entangled partner.

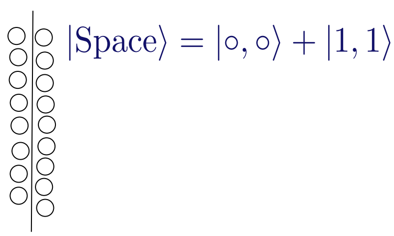

Figure 2: Partition of space into cells. In each cell there may or may not be a particle. Because the cells are sufficiently small, there is a high degree of finding another virtual particle across the partition if one is found on the other side. Hence, the correct description is the one shown in the wave function, Eq. (**??**).

depicted in Fig. (**??**). Note this type of entangled state seems some what unlike the state with $\otimes$ and $\hat{\otimes}$ with particle occupancy being the relevant variable. Nonetheless, it is exactly of the canonically maximally entangled form. As is dictated by quantum theory, any measurement of what is in the cells destroys the entanglement. Measurement requires energy which has gravitational effects. Once the entanglement is destroyed, the space is not tethered together–that is, it can be unzipped. Hence, entanglement is a crucial ingredient for the contiguous structure of spacetime. We are beginning to see that even spacetime abides by quantum mechanics. Spacetime is literally held together by entanglement.

Consider now a black hole shown in Fig. (**??**). A black hole has a horizon–that is a point of no return. Consider two individuals Bob and Alice. Lets send Bob through the horizon. He makes a measurement and finds a particle just inside the horizon. Since the horizon is smooth, it shares the properties of empty space as depicted in Fig. (**??**) and hence it consists of a teeming collection of virtual particles completely

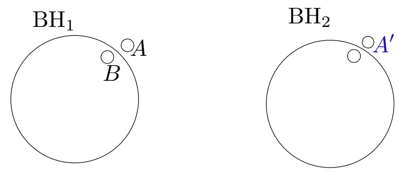

Figure 3: Two entangled black holes. By construction A and B are entangled in BH_1 which is entangled with BH_2. The problem arises that A' is entangled with A as well thereby violating the principle of monogamy of entanglement.

entangled. So we reach the conclusion that the particle that Bob measures behind the horizon must be entangled with something on the outside. Call these two pieces of information B and A as depicted in Fig. (**??**a). Now suppose that BH_1 is entangled with BH_2 by the mechanism we used to construct two macroscopically entangled bins in Fig. (**??**). We can also construct BH_2 from the evaporation of BH_1. By evaporation, we simply mean Hawking radiation, black body radiation consisting of particle-antiparticle pairs. Here's the problem: particle A violates monogamy of entanglement. That is, A is entangled with B but also with A' by virtue of the entanglement between BH$_1$ and BH$_2$, which violates the principle of monogamy of entanglement. How can this be since monogamy is prohibited by quantum mechanics.

AMPS proposed as a resolution that the properties behind the horizon of a black hole look fundamentally different than what we had previously thought. Namely, there can be nothing in the black hole–just a firewall. This firewall prevents anything from being in the interior of a black hole. Note by nothing we do not mean empty space because that would be something quantum mechanically. This seems to violate the equivalence principle of Einstein's that no local measurement can tell you that you are in a uniform gravitational field. So their resolution involves giving up on one of the fundaments of GR. Is this the only option available to them?

An alternative that they rejected is that the particle outside BH_2 which we call A' is actually B. AMPS rejected this because they assumed there is no causal connection between BH_2 and BH_1. That is, the distances are too large for them to be causally connected. Lets say that Alice is in charge of BH_2. She makes some measurement on A' which amounts to a measurement on A and hence also on B since A and B are entangled. Measurements cost energy. So it seems that there is much energy being transferred at quite far distances. Further, whatever particle Alice chooses to measure there must be an analogous pair behind the horizon in BH_1 since the two black holes are entangled. They thought this to be unlikely and hence

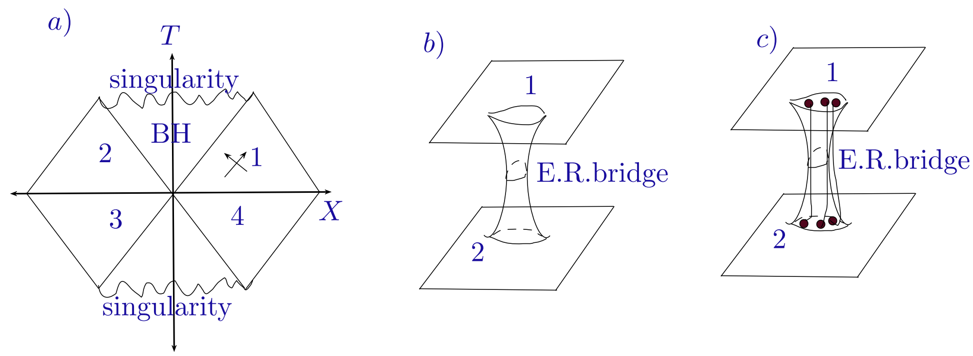

Figure 4: Kruksal coordinate representation of a Penrose diagram of a blackhole. Light is sent only at 45^o. As a result regions 1 and 2 are not causally connected. The region between 1 and BH is the horizon. Notice there is another horizon bordering 2 and BH as well. These regions are disconnected as depicted in (b) but information can flow between them through a wormhole, an Einstein-Rosen bridge.

settled on the firewall as the resolution of the paradox.

Their rejection of this solution is based on a key assumption: No causal connection exists between the bits of information in BH_2 and the stuff behind the horizon in BH_1. Let's explore if in fact it is reasonable that $A' = B$ is true. What this would mean is that stuff outside BH_2 is actually stuff inside the horizon of BH_1. This seems crazy but is actually true. To show this, we need to construct a spacetime diagram that is suitable for looking at black holes: Kruksal also known as a Penrose diagram. Depicted in Fig. (**??**a) is a spacetime diagram of a black hole. Regions 1-4 are outside the black hole and the borders (horizon) of the black hole are the lines separating 1-4 from BH. Hence, there are actually 4 black holes shown as there are 4 distinct horizons. Note, light signals, which must propagate at 45^o (as depicted), cannot be sent from region 1 to 2. Hence, they are disconnected regions of spacetime. Consequently, the spacetime between 1 and 2 is empty. We have learned that such empty regions are entangled. Here in lies the nub of the argument. Such disconnected regions of spacetime are connected by a construction called an Einstein-Rosen bridge which J. A. Wheeler termed a wormhole. That is, particles leaving 1 at a 45^o can intersect particles leaving region 2 (also moving at 45^o) through a bridge. Such bridges represent the entanglement of regions 1 and 2. What this implies is that a particle outside 1 can end up inside 2 because 1 and 2 are connected by a wormhole. That is, it is entirely reasonable that A′ is causally connected with B. In fact, this is what E.R. bridges allow: information outside BH_2 can end up inside BH_1. Consequently, a possible resolution of the paradox is that $A' = B$. Consequently, there is no problem with monogamy of entanglement.

The key thing here is that enough entanglement in a small enough volume can manifest itself as a new region of spacetime. Hence, the topology of spacetime is much richer than originally thought. It is not just about curvature. Quantum mechanical entanglement is at the heart of it. Fig. (**??**c) illustrates this. Take a bunch of entangled stuff. Compress the particle lines to form a black hole. The vertical lines coalesce creating a new region of spacetime. Topology of space is more complicated than originally thought. Entanglement builds spacetime

The criticism of this argument is that this resolution is a bit unfulfilling in that you can never extract the information from a black hole because of the necessity of a worm hole.