Lecture 26: April 27, 2021

PHYSICS 419 - Spring 2021

1 Inflationary Cosmology

Combining SR and QM requires that the number of particles of some type be represented by a quantum field. It is therefore subject to uncertainty relations. As we saw last class, treating the vacuum as if it were filled with a sea of particle-antiparticle pairs blinking in and out of existence, giving a "zero-point energy" leads to entanglement of regions of spacetime.

This treatment is not just hypothetical. The vacuum energy in the E-M field depends on confinement by conductors. Therefore a force is exerted on the conductors (the Casimir force). It's measurable rather precisely.

Remember, a force depends on differences in energy as a function of position. So measuring the Casimir force, etc, does not tell you what the energy density is of empty space, only how it changes when pieces of metal, etc, are moved around in it.

Absolute energies (as opposed to differences) enter into physics as the masses which give rise to gravity. So we have to ask whether we can calculate the energy density in space that serves as a source of gravitational effects simply by adding up the sorts of background energies that give rise to the Casimir effect.

If we do so, we end up with an energy density some 10^{125} times as big as the critical density of cosmology.

What is Symmetry?

There are two ways of viewing symmetries: 1) ontological facts of nature. On this account, symmetries are a substantial part of the natural world. That is, symmetries represent properties existing in nature. They in fact lead us to new predictions, for example a particle physicist's use of symmetry to predict new particles. 2) The epistemological account of symmetries emphasizes that symmetries ultimately are related to our ignorance. That is, rotational symmetry of a sphere prevents us from picking one point of a sphere as opposed to another and calling it distinct. Symmetries are associated with unavoidable redundancy in our descriptions of the world (SEP). Equivalently, symmetries render certain types of descriptions of an object unintelligible, for example distinct points on a sphere.

Spontaneous Symmetry Breaking

You may recall that the conservation laws follow from symmetries that the universe has. For example, angular momentum conservation follows from the invariance under rotations of the equations. The important point here is that the state of lowest energy doesn't necessarily have to exhibit the symmetry.

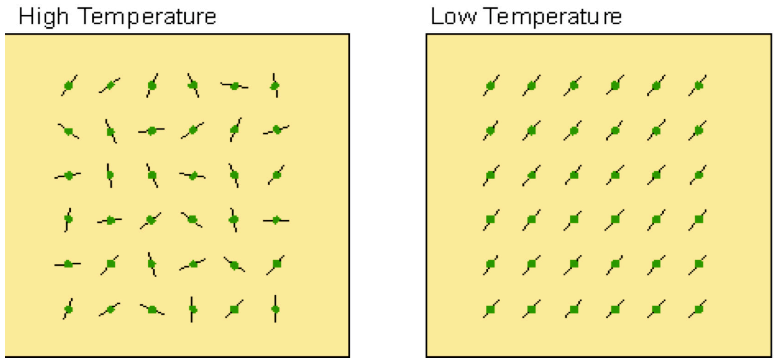

Figure 1: Spin system at high and low temperature. At high temperature, the spins are disordered, pointing in all directions. At low temperature, they order and the system develops a bulk magnetization.

Let's consider an everyday example of a piece of iron. We're interested in the interior of the iron, not the shape of its surface. Each atom behaves like a little magnet, which we'll call a spin. What we want to know is, "Will the spins line up with each other?" The answer depends on the temperature of the iron:

At high temperatures, the spins point in random directions. That's because there are many more quantum states with random-looking spins than there are with the spins in any simple pattern. The iron is not magnetized-that is, oriented and, on average, the material is isotropic. The spins have lower energy when they line up. At low T, that energy can make more entropy off in the nearby reservoir than the spins lose by lining up. When the iron is cooled below about 770^oC (the Curie temperature), the spins do align, maximizing the TOTAL entropy. The iron becomes magnetized.

However, they cannot align without picking some particular direction to align along. Now there is a preferred direction, even though in essence, the physical equations contained no special direction. This process is called spontaneous symmetry breaking. Rotational symmetry is broken in this example.

If the symmetry is broken, how do we know it was ever there? A remnant of the original symmetry is always left behind. The tell-tale sign of spontaneous symmetry breaking is a pair of situations, one having more symmetry than the other, which share many properties. Symmetry breaking is described by a parameter whose value is zero when the symmetry is restored. Consider our magnetized iron. At low temperature, the iron is not isotropic. There is a preferred direction. Nevertheless, rotations around the magnetization axis leave the iron unchanged, so some of the symmetry remains. (Remember that we are ignoring the surface.)

The strength of the symmetry breaking is described by the magnetization. As that quantity goes to zero, the symmetry is restored.

This spontaneous symmetry breaking has various effects.

Depending on the details of the symmetry broken, there can be a variety of new types of excitations around the broken-symmetry state. Goldstone's theorem states that anytime a continuous symmetry is broken a massless excitation comes about, termed a Goldstone boson. The Higgs particle is one such example.

In the context of magnetism, the massless mode is a magnon in which the spins twist a bit from the preferred orientation, and spring back due to torques from neighboring spins.

There can be boundaries between regions which break the symmetry in different ways.

Other symmetries can spontaneously break:

On the average, any position in a liquid is just like any other position. When the liquid cools, it can freeze into a solid, in which there is a distinction between the sites with atoms on them and the spaces in between. Spatial translation is the broken symmetry. Some sugars (and other big molecules) have left-handed and right-handed forms, which twist polarized light opposite directions. Crystals form which contain only one type or the other. Within the crystal, PARITY (mirror-image) symmetry is broken. (If the interconversion rate between the two forms is big enough, all the molecules can end up in one crystal, breaking parity for the whole batch.)

We saw that physical laws can have various other symmetries, besides these simple spatial symmetries. E.g. time-reversal, Lorentz transformations.

Any underlying symmetry can be broken by the equilibrium physical state, just as the spatial symmetries were broken in the examples above.

Gauge symmetries are special in the sense that they arise from statements about mathematical invariance of the underlying theory. Charge conservation is a gauge symmetry.

Let's imagine a universe where every particle is massless. Suppose there is a particle (call it h, for Higgs) which interacts with the other kinds of particles and with itself.

The fact that the h particles interact with each other can give rise to interesting and unusual results. Consider a box containing these particles. How does the energy of the box vary as we increase the density of h particles in the box? Normally, we would expect it to increase:

The graph is not linear, because the particles interact.

Figure 2: Energy versus density for some field theory.

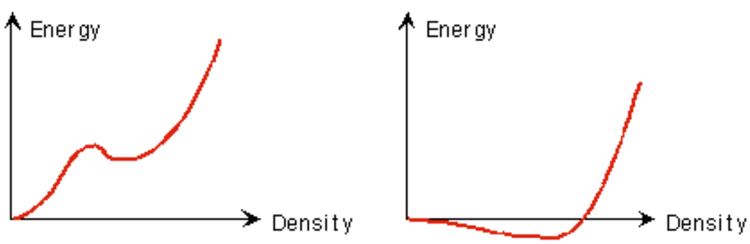

Figure 3: False vacuum (first figure) and inflationary scenario (second).

Suppose the h particles have an attractive interaction such that above a certain density they begin to condense. Then the energy vs. density graph will look like this:

On the right, the state of lowest energy has nonzero density!

There is nothing terribly exotic about these pictures of energy vs density. Essentially identical pictures occur for ordinary gasses, when you try to calculate whether they form dense liquids or stay as rarified gas. The difference arises in that we treat the gas molecules as coming from some reservoir with a fixed number of particles. Here, in contrast, we're dealing with particles which can pop in or out of existence. Empty space itself provides the reservoir, and can never be emptied of the potential to produce more of the particles.

In equilibrium, the total entropy is maximized, which is equivalent to minimizing the total free-energy. The density (strength) of the Higgs field is zero at very early times, but as the temperature decreases, that is no longer the favored configuration. If the middle energy-density graph is correct, then the universe will

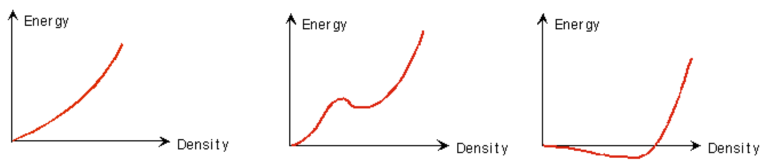

Figure 4: All three scenarios.

become trapped in a false vacuum state, i.e. a state in which the free-energy is bigger than it has to be. Even if the last version is right, the universe will not instantly reach the true vacuum, but will spend a little time in false vacuum states. The false vacuum will cause the expansion of the universe to have unusual features.

(See Scientific American, Jan. 1999 for excellent articles describing the difference between getting stuck for a while in a false vacuum [middle picture] and gradually rolling out of a false vacuum [last picture], and for a cosmological update.)

As a gas expands, its density decreases, and it does work (PdV) so its energy also decreases. This is not the case with the excess energy due to the false vacuum. The energy density remains constant as the universe expands, so the energy in an expanding volume actually increases. That is, the vacuum will act like a gas with negative pressure.

As long as the universe remains in the false vacuum, it will expand exponentially. This is known as inflation. It is speculated that inflation occurred about 10^{-33} sec after the big bang.

Successes of Inflation

1.) The homogeneity of the universe. Exponentials become large very fast. If the inflation lasted for 100 time constants, the universe expanded by a factor of $e^{100} = 10^{43}$. The entire visible universe (10^{25} m) was a basketball (30 cm) just after inflation, but only a minute speck (10^{-43} m) just before. This is much smaller than the causally connected region (10^{-23} m) at that time.

2.) The density of the universe. Suppose the universe wasn't anywhere near the critical density before inflation. This means, in the balloon analogy, that it was curved. Now, increase the radius of the balloon by 10^{43}. It will become indistinguishably close to a flat surface.

What's Wrong With Inflation?

There is no evidence for the proposed interactions, which require the existence of a new, previously unobserved, force. Inflation does make predictions for the amount of structure of various sizes in the universe, especially in the background radiation.

Also, why would inflation stop. Some think it didn't and other universes sprouted off of this one and hence other big bangs are out there. How is this falsifiable? Remember what distinguishes a scientific theory from everything else is not its agreement with experiment but by its falsifiability.

Another problem is that quantum effects predict that the universe coming out of the inflationary epoch would not ultimately be smooth nor isotropic but rather patchy. So the very problem it was invented to solve, it completely messes up. So why believe inflation?

So we have strong reason to think that inflation made the total energy density very close to the critical value. In fact, if the same inflation that made the universe homogeneous also made it flat, the radius of curvature should be bigger than 100 as large as the age of the universe times c. That would mean that the universe is extremely close to flat: right at the edge between open and closed.

Does that mean that we are missing 70% of the matter in the universe, when we only find 30% of the critical density in galactic clusters?

No: the expansion of the universe is STILL ACCELERATING according to pretty reliable measurements.

In other words, it looks like we're still in a period of very weak inflation!

Once you allow for background energy density, causing accelerating expansion, the simple connection between energy density and the fate of the universe is broken.

Depending on the ratio of the ordinary mass to the background density, you can have:

Open/ always expands

Closed/Collapses

Closed/always expands

Open/collapses

We seem to be almost exactly on the edge between open and closed, very near flat.

If the field apparently causing the current acceleration of expansion remains constant, the expansion will continue forever. If it were to fall to zero, the expansion would still continue forever. But we really don't know what it is or what it will do.

Cosmological implications

Consider the early, hot universe. Suppose that there is a Higgs field which behaves as I described above. Then, as time progresses, the free-energy-density graph (a measure of how the net entropy depends on the variable) will evolve: