Lecture 24 - Cryogenics

Cooling usually involves a working substance (ex: gas, free e^- gas in a solid, ^3He in ^4He, etc.)
Two common principles for cooling are: using expansion of gas and evaporative cooling

Expansion engine

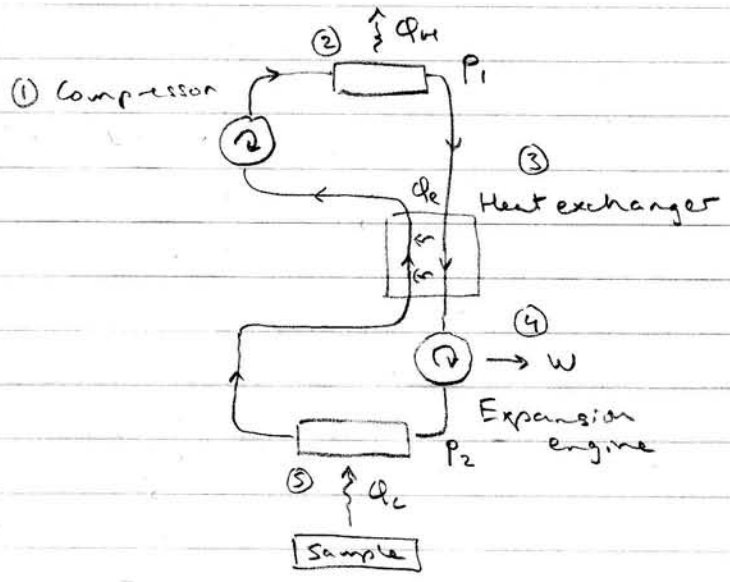
During isentropic expansion of a gas, we showed

$$pV^\gamma = \text{const.} \qquad \gamma = \frac{5}{3} \text{ for ideal monatomic gas}$$

$$p\,T^{\gamma/1-\gamma} = \text{const.} \qquad \text{since} \quad pV = Nk_BT$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-1/\gamma} = \left(\frac{p_2}{p_1}\right)^{2/5}$$

If gas is expanded $p_2 < p_1$ and $T_2 < T_1 \Rightarrow$ cooling
Usually expansion is done with a turbine, in a cycle:



① compression of gas
② Heat of compressed gas ejected into environment
③ Heat exchanged with cool return gas
④ Isentropic expansion of gas ⇒ cooling
⑤ Cold gas extracts heat from sample

Expansion is isentropic, so no heat exchanged, but work is done by the gas:

$$W = (U_1 + p_1V_1) - (U_2 + p_2V_2) = H_1 - H_2$$

$U_1 + p_1 V_1$ = internal energy of incoming gas + work done by compressor

$U_2 + p_2 V_2$ = internal energy of outgoing gas + work to move gas against pressure p_2

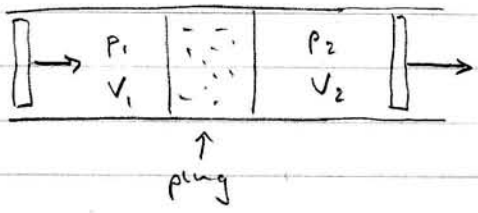
$H = \frac{3}{2} N k_B T + N k_B T = \frac{5}{2} N k_B T$ for ideal, monatomic gas

$W = \frac{5}{2} N k_B (T_1 - T_2)$ $\quad (T_1 > T_2)$

This is an efficient way of cooling gas. However, can't be used to liquefy gases because of mechanical difficulties with turbine in expansion engine.

Throttling (or Joule-Thomson effect)

This method used to liquefy gases. Only requires porous plug or nozzle (i.e. no moving parts) to maintain pressure differential $p_1 > p_2$



Here, no net work is extracted by expanding gas

$\therefore W = 0 = (U_1 + p_1 V_1) - (U_2 + p_2 V_2)$
$= H_1 - H_2$

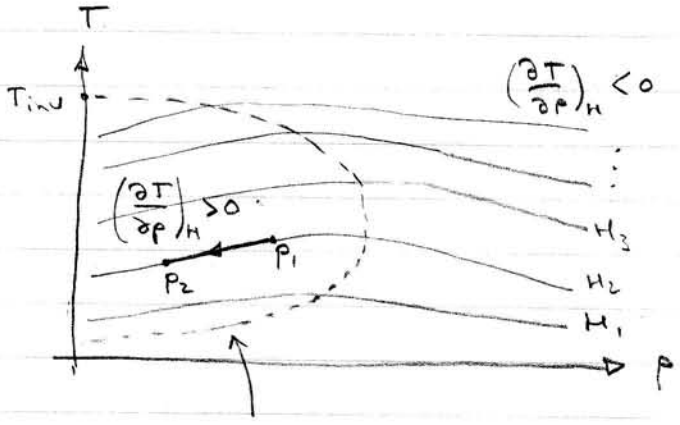
$\therefore H_1 = H_2$ constant enthalpy gas expansion

For an ideal gas $H = \frac{5}{2} N k_B T$ so constant H means constant T $\Rightarrow$ i.e. no cooling

However, for a real gas (i.e. van der Waals gas) H will depend on p, T. Will get cooling as long as:

$$\left(\frac{\partial T}{\partial p}\right)_H > 0$$

Plot lines of constant H on p-T plane for van der Waals gas



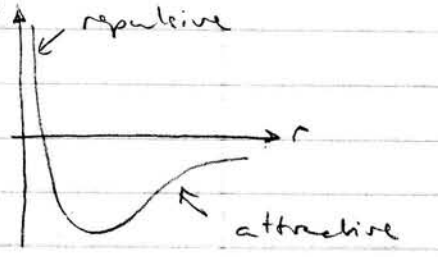
Dotted line is the inversion curve, where sign of $\left(\frac{\partial T}{\partial p}\right)_H$ switches

In region inside curve expansion will cool gas

Why do we need van der Waals model to explain this?

In ideal gas, U, H are independent of gas density n or pressure p

In van der Waals gas, there are long-range attractive forces (weak) and short-range repulsive forces (strong)



change in sign of $\left(\frac{\partial T}{\partial p}\right)_H$ has to do with whether attractive or repulsive component dominates

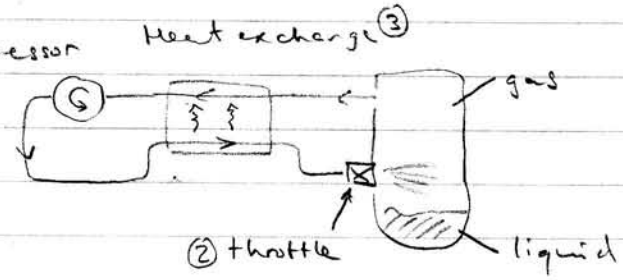
$T > T_{inv}$ Kinetic energy of gas is so high, attractive forces are negligible but repulsive forces are not. Expansion = decrease in P.E.
gas does $-W$ and T increases $\therefore \partial T/\partial p < 0$

$T < T_{inv}$ K.E. is low, attractive forces dominate.
Expansion = increase in P.E., gas does $+W$ and T decreases.
$\therefore \partial T/\partial p > 0$

$T_{inv}(N_2) = 621\,K$ $T_{inv}(H_2) = 205\,K$ $T_{inv}(^4He) = 51\,K$

$\therefore$ To liquefy ^4He, first need to precool to $T < T_{inv}$. Can use liquid N_2 (77K)

Liquefaction of gases commonly done in Linde cycle

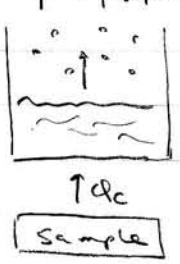


① Compressor pushes gas through throttling valve
② Gas cooled by throttling Some is liquefied, vapor pumped back to compressor
③ Heat exchange between hot incoming gas and cool outgoing gas

Liquid N_2 = 77K at p = 1atm
^4He = 4.2K

Evaporative cooling

There is a latent heat with evaporation $L = T\Delta S$



If you pump away vapor above a liquid, so p < vapor pressure, then molecules leave liquid phase ⇒ evaporation.

Requires latent heat L, supplied by Q_c from sample ⇒ cooling

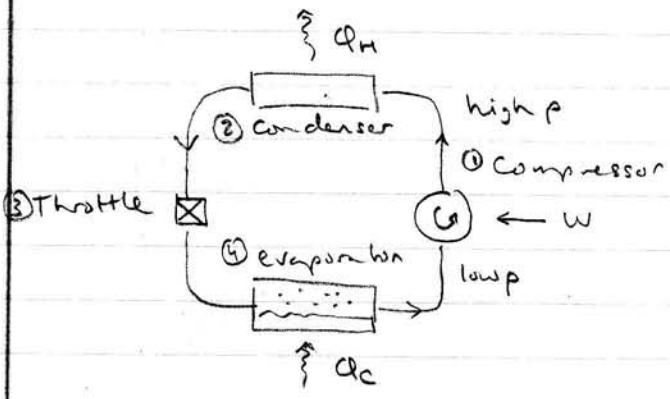
Generally $\frac{dQ_c}{dt} = L\frac{dN}{dt}$ $\frac{dN}{dt}$ = rate of removing molecules in vapor phase

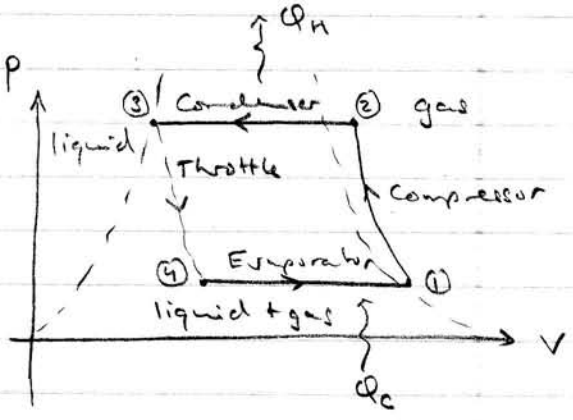
As T decreases, so does p_{vapor}. (Recall $p_{vapor} = p_0 e^{-L/RT}$)
so we need to pump harder to get $p < p_{vapor}$

For ^4He, practical limits to pumps take T from 4.2 → 1.2K

^3He is a lighter isotope of ^4He. Turns liquid at 3.2K, has a higher vapor pressure, so it can be cooled further by evaporative cooling. Practically 3.2 → 0.3K

Modern refrigerators use throttling + evaporation to cool.
Working substance is freon or hydrofluorocarbon
($T_{boil} = -29°C$ and $-47°C$)

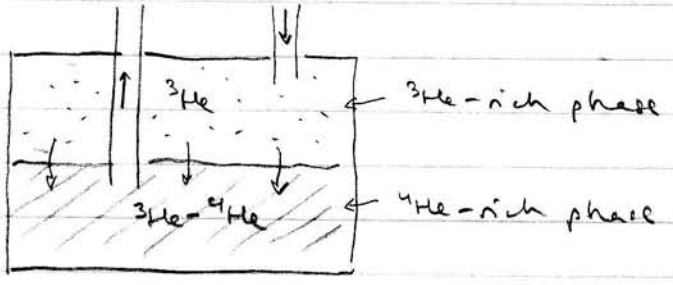




① Compression of gas raises T, p (isentropically)
② Hot gas expels heat Q_H to environment, condenses to high T, p liquid
③ Throttling cools liquid lowers pressure
④ Evaporation converts liquid to gas, extracts Q_c from cold reservoir (i.e. refrigerator or freezer)

Dilution refrigerator

Preferred method for cooling below ~0.3K since 1960's
Uses mixture of ^4He, ^3He. Below 0.87K, ^4He and ^3He are immiscible ⇒ mixture phase separates into a ^3He-rich phase (essentially pure ^3He) and a ^4He-rich phase (has 6% ^3He). ^3He-rich phase is lighter ⇒ floats on top



^4He is essentially inert
^3He is a fermi gas

Principle of cooling is similar to evaporation
As ^3He molecules move from top ^3He-rich phase to bottom ^4He-rich phase, they are diluted and gain entropy $\Delta S = S(^4He\text{-rich}) - S(^3He\text{-rich})$

This is like a latent heat $L = Q_c = T\Delta S$ extracted from cooling sample

^3He in ^4He-rich phase acts like vapor, ^3He-rich phase as liquid.

To get constant rate of cooling, need to dilute ^3He in ^4He-rich phase below 6%, then ^3He will evaporate into ^4He

Dilution cooling works from $\sim 0.5K \rightarrow 10mK$

Isentropic demagnetization

This relies on paramagnetic properties of working substance
Consider N spins (electronic or nuclear) in a B-field

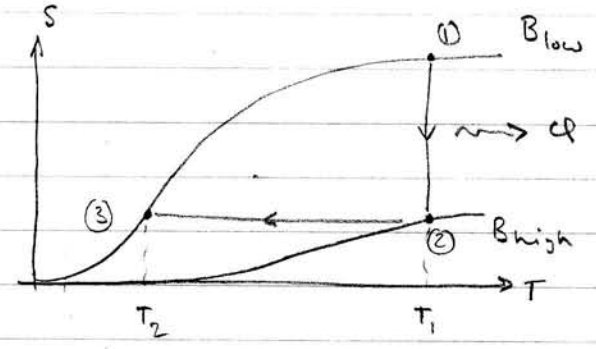
$$Z_1 = e^{\mu B/k_B T} + e^{-\mu B/k_B T} = 2\cosh\frac{\mu B}{k_B T}$$

Showed in homework

$$S = N k_B \left[\ln\left(2\cosh\frac{\mu B}{k_B T}\right) - \frac{\mu B}{k_B T}\tanh\frac{\mu B}{k_B T} \right]$$

Notice S is a function of $\frac{B}{T}$: $S\left(\frac{B}{T}\right)$

Plot S for 2 B-fields B_{high}, B_{low}



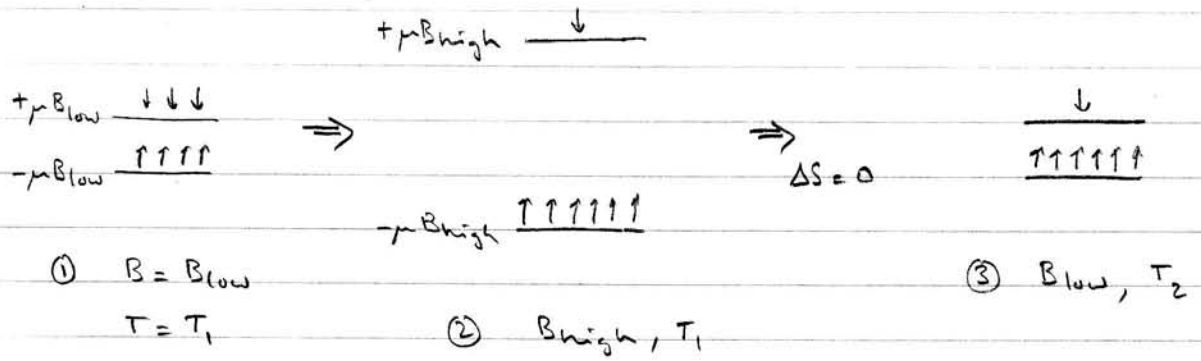
①→② Increase B ($B_{low} \rightarrow B_{high}$) isothermally. Spins become more ordered, S decreases (Q out)

②→① Decrease B isentropically system cools from $T_1 \rightarrow T_2$

Since $S = S\left(\frac{B}{T}\right)$ $\qquad \frac{B_{high}}{T_1} = \frac{B_{low}}{T_2}$

$\therefore \quad T_2 = T_1 \frac{B_{low}}{B_{high}} \ll T_1$

Energy level diagram



① $B = B_{low}$, $T = T_1$ ② B_{high}, T_1 ③ B_{low}, T_2

During second step, relative populations are maintained ($\Delta S = 0$)
$\therefore$ T decreases

With this approach can get to 1mK for electronic paramagnets
1μK for nuclear paramagnets!

Why is there a limit? If you make $B_{low} = 0$, shouldn't $T_2 = 0$?
No! Interactions between spins are important below a certain T
Get ferromagnetic ordering. As we saw for Weiss model,
interactions are like residual B-field $\therefore$ $B_{low} \gtrsim 0$ practically