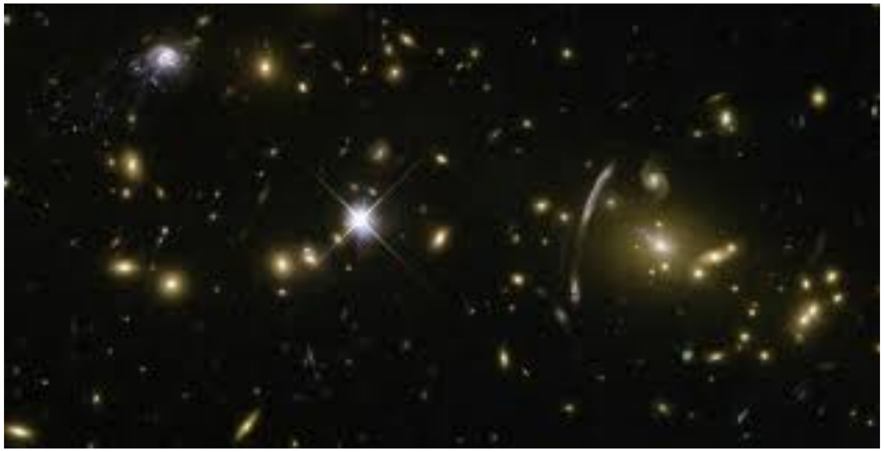
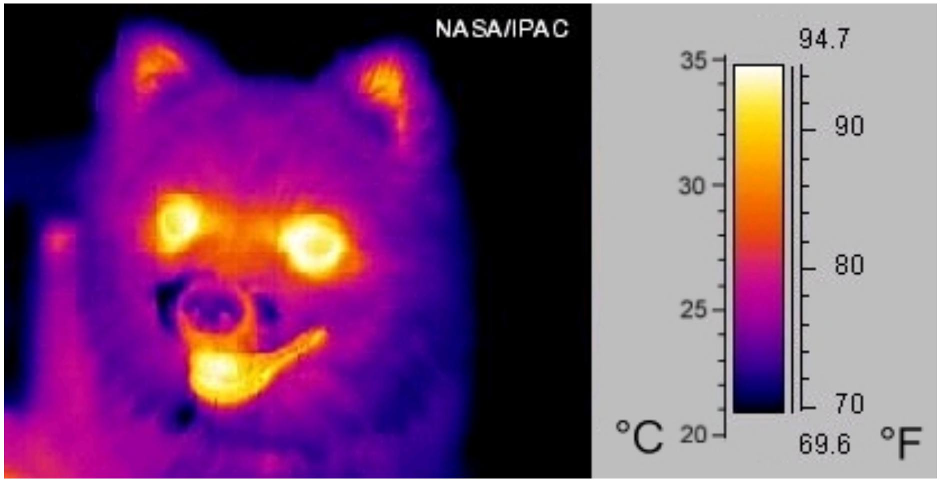
Physics 435

Electromagnetic Fields I





Physics 435

Electromagnetic Fields I

All course information is here:
http://courses.physics.illinois.edu/phys435/fa2013/

This includes:
- Course syllabus
- Homework assignments and solutions
- Discussion problems and solutions
- These lecture slides (pdf)
- Announcements

Also, course policies and procedures.

There is a discussion session tonight (math review)

There is a homework assignment due this Friday!!

(two math review problems).

Course components:

- Reading. Do it before lecture. Books: *Griffiths* and *Feynman.* Each reading assignment (on the syllabus) is short.
- Homework. Important!! Learn by doing. 35% of your grade
- Discussion. Short, often conceptual, problems. 10%
- Two midterm exams. (to keep you honest) 15% each
- Final exam 25%
- Lectures . Examples and difficult concepts. 0% (but come anyway)

  I hope we can interact. I don't want to be the only one talking.
  Please ask questions !!!

Goals:

- Learn E&M, building on P212.
- Learn how this fits into the bigger picture (mechanics, QM, *etc*.) Electrodynamics is classical field theory. Many concepts and techniques carry over to QM.

People

			Office hours:
• Me:	Jon Thaler	333-8174	Thursday 5-7 PM
		jjt@illinois.edu	158 Loomis
		427 Loomis	
• TAs:	Discussion:	Peter Sahanggamu	Wednesday 3-4 PM 390C Loomis-Seitz Interpass
	Homework:	Xueda Wen	aaa 4111 ESB
		Chun Kit Chan	ccc 271 Loomis

I will grade exams.

The Plan for the Next Few Lectures

- Some Mathematical preliminaries:
  - Transformations and invariants. Galilean invariance.
  - Dot and cross product
  - Flux
- Electric field
  - Divergence and curl
  - Electric potential
- Work and Energy
  - Self-energy of a collection of charges
- Conductors in Electrostatics
  - Constraints and boundary conditions
  - Images
  - Capacitance

Then, Poisson's and Laplace's equations.

This Week's Readings

I will not normally announce them

- Today: Griffiths: "Advertisement" pp. *ix-xv*.
  Appendix C (units), pp. 558-561.
  Feynman: Ch. 2, 3

- Wednesday: Griffiths: Ch. 1, Ch. 2.1
  Feynman: Ch. 2, 4-1

- Friday: Griffiths: Appendices A and B
  Feynman: Ch. 2, 3

Each day's reading is only a few pages.
Please do the reading before lecture.
You'll get a lot more out of lecture.

A Tiny Amount of History

The development of E-M is largely a 19th century story.

1820:	Oersted	Electric currents create magnetic fields.
1831:	Faraday	A changing **B** field produces an EMF.
1864:	Maxwell	The complete theory.
1888	Hertz	Electromagnetic radiation.

Maxwell's theory unified electricity and magnetism (and optics, chemistry, *etc*.). This is conceptually similar to:

17th century:	Newton unified terrestrial gravity and solar system dynamics.
20th century:	Weinberg, *et al*. unified E-M and the weak nuclear force.

Simplicity of description remains physics' greatest driving force.

Physicists have few principles,
and hope to have fewer.

Units (Appendix C)

There are two important systems of units in E-M:

SI (rationalized mks): $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Coulombs
meters
ε_0 = C^2/N m^2

This is what we all know and love from Physics 2xx. Volts, amps, etc.

Gaussian (cgs): $$F = \frac{q_1 q_2}{r^2}$$

esu
centimeters
ε_0 is gone!

Gaussian units are more suited to pure (not applied) study. No weird constants. Also (as we'll see), **E** and **B** have the same units in the Gaussian system. It also makes the symmetries of E-M more explicit. So, why aren't it used? Historical: The practical units were developed before the symmetries were known.

We'll use SI units except in parts of 436,

when special relativity and radiation make Gaussian units preferable.

Mathematical Preliminaries (1)

Griffiths says, "Charge comes in two varieties." This is misleading. He's referring to ± charge. More correctly, "Electric charge is a real number." It can take any positive or negative value. (We ignore quantization in this course.)

Two important properties of charge:

- It is a property of the object. That is, q does not depend on the environment.
- It is a scalar. It is invariant (does not change) under a coordinate rotation or other change of reference frame.

Mathematical Preliminaries (2)

Some physical quantities (*e.g.*, ***E***), are vectors.

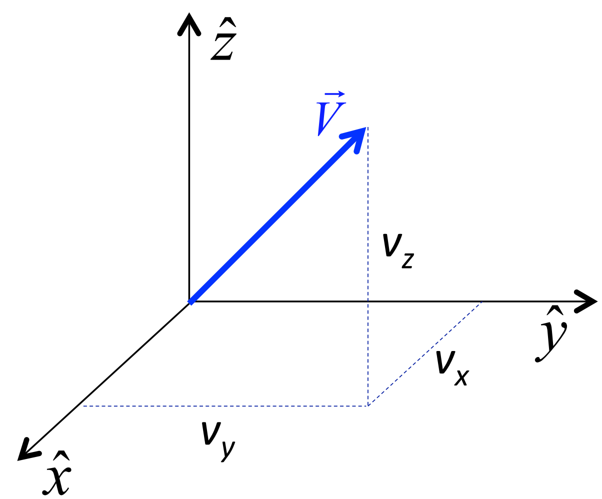
This implies:

- One needs three numbers (components) to specify them.
- Vector components transform in a specific way under rotations.

You should get used to performing vector operations using components.

Notation:

- ^ denotes a unit vector: $\hat{x}$
- For PowerPoint simplicity, I will often use bold face to denote vectors: ***V***.



$$\vec{V} = v_x\hat{x} + v_x\hat{y} + v_z\hat{z}$$

Mathematical Preliminaries (3)

Dot (inner) product: $\hat{x}\cdot\hat{x}=\hat{y}\cdot\hat{y}=\hat{z}\cdot\hat{z}=1$

$\hat{x}\cdot\hat{y}=\hat{x}\cdot\hat{z}=\hat{y}\cdot\hat{z}=0$

Orthonormal coordinates

So: $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z=\vec{B}\cdot\vec{A}=|A||B|\cos\theta_{AB}$

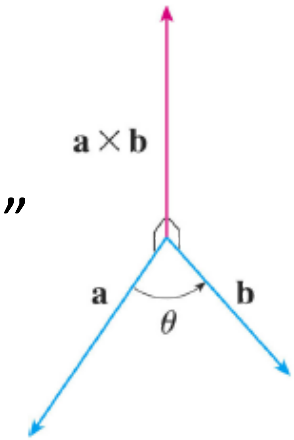
$\vec{A}\cdot\vec{A}=|A|^2=A_x^2+A_y^2+A_z^2$ Pythagoras' theorem

Cross (outer) product: $\hat{x}\times\hat{y}=\hat{z}$

$\hat{y}\times\hat{z}=\hat{x}$

$\hat{z}\times\hat{x}=\hat{y}$

"Right handed" coordinates



Also: $\hat{A}\times\hat{B}=-\hat{B}\times\hat{A}$ The cross product anti-commutes.

So, $\hat{A}\times\hat{A}=0$

Finally, $|\vec{A}\times\vec{B}|=|A||B|\sin\theta_{AB}$

Exercise: Using this (the definition of cross product): Show that $(\vec{A}\times\vec{B})\cdot\vec{A}=(\vec{A}\times\vec{B})\cdot\vec{B}=0$. That is, the result of the cross product is a vector perpendicular to both operands.

Mathematical Preliminaries (4)

For completeness: (you'll see this a lot in later courses)

Define the Levi-Civita tensor, ε_{lmn}:

$\varepsilon_{123} = 1$

$\varepsilon_{lmn} = +1$ if *lmn* is an even permutation of 123.

$\varepsilon_{lmn} = -1$ if *lmn* is an odd permutation of 123.

$\varepsilon_{lmn} = 0$ if any two indices are equal.

Explicitly:

$$l=1:\quad m\overset{n}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & +1 & 0 \end{pmatrix}} \quad l=2:\begin{pmatrix} 0 & 0 & +1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad l=3:\begin{pmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Then, we can write: $\left(\vec{A}\times\vec{B}\right)_l = \varepsilon_{lmn} A_m B_n$

where I used the Einstein summation convention,

Repeated indices (in products) are summed over: $A_m B_m \equiv \sum_1^3 A_m B_m = \vec{A}\cdot\vec{B}$

Invariance Principles

Transformation of vectors under rotation of coordinates.

This is the important property of vectors.

Q: Why are the transformation properties important?

A: This is due to the empirical fact that one cannot tell what reference frame we are using. This is Galilean Relativity (or invariance).

> The laws of physics must look the same in all reference frames that are related by rotations, translations, and constant velocity boosts.

Invariance properties are most easily enforced by writing the equations to be manifestly covariant.

Example: Suppose we have: $Z = aX + bY$.

a and b are constants.
X, Y, and Z are variables.

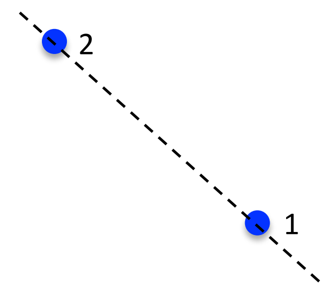
We can guarantee rotational invariance by enforcing:

- If Z is a scalar, then both aX and bY must be scalars.
- If Z is a vector, then both aX and bY must be vectors.

This approach is used whenever invariance principles are known. *E.g.*, SR, GR, and particle physics.

Invariance Example

Consider two point particles that are described by their masses (m_1 and m_2) and electrical charges (q_1 and q_2). What force does 1 exert on 2?



Because ***F*** is a vector, we must construct a vector from the quantities at hand. The only vectors we have are the particle positions, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. Thus, we must have: $\vec{F} = a\vec{r}_1 + b\vec{r}_2$
where a and b are scalar functions of m_i, q_i and $\boldsymbol{r}_i$.

We can also impose translational invariance. The only vector that remains unchanged by a translation is: $\vec{\mathscr{r}} \equiv \vec{r}_1 - \vec{r}_2$

Thus: $\vec{F} = k\vec{\mathscr{r}}$

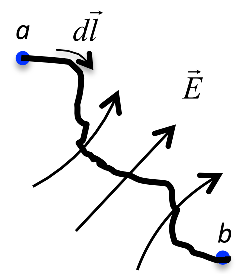
The force must point along the line joining the two particles. Can you think of a counterexample?

Note: (added after lecture)
$\vec{r}_1 \times \vec{r}_2$ is a pseudovector and is not allowed. The universe is observed to have mirror symmetry, but pseudovectors (and pseudoscalars) change sign under mirror transformations.

NOTE: All physical constants must be scalars!! (Why?)

More Math

Line and Surface Integrals: (Griffiths, 1.3)
These are the most ubiquitous calculations in E-M.



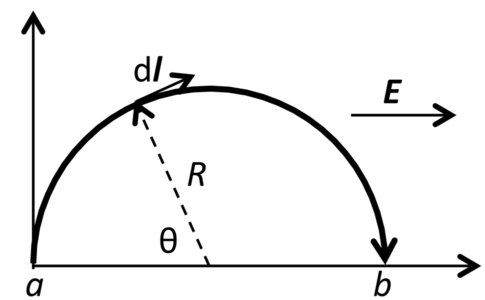
Line: $\int_a^b \vec{E} \cdot d\vec{l}$

This tells us the potential difference, $\Delta V_{ab} = V_a - V_b$, because the energy gained by a charge as it moves from *a* to *b* is:

$$\text{Work} = \int_a^b \vec{F} \cdot d\vec{l} = q\int_a^b \vec{E} \cdot d\vec{l} = -\Delta PE = -q\left(V_b - V_a\right)$$

To evaluate the integral, we need to express ***E***·d***ℓ*** as a function of the position along the path. This is a 1-dimensional problem.

Example: A charged particle moves along a semicircle in a uniform field, ***E*** = (E_x, 0).



$$d\vec{l} = \left(dx, dy\right) = \left(\sin\theta, \cos\theta\right) R\,d\theta$$

The position along the path can be described by a single variable.

$$\vec{E} \cdot d\vec{l} = E_x R \sin\theta\, d\theta$$

$$V_a - V_b = E_x R \int_0^\pi \sin\theta\, d\theta = 2RE_x$$

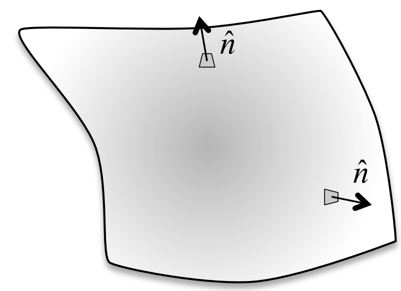
This is the same answer as if the particle had moved in a straight line, as expected (I hope).

More Math (2)

Surface integral: $\int \vec{E} \cdot \hat{n}\, dA$

We are often interested in the flux through a surface.
This is used in Gauss' law and also for inductance calculations (G, chapter 7).

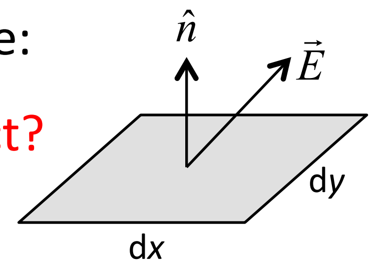
The flux is calculated using a vector normal (perpendicular) to the surface.
On a curved surface, the direction of ***n*** will vary with position.



The flux of ***E*** through an infinitesimal surface:

$$d\,\text{Flux} = \vec{E} \cdot \hat{n}\, dA$$

Why the dot product?

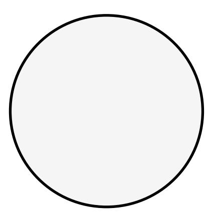


Comments:

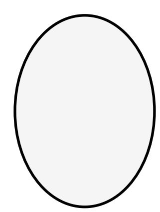
- This is a 2-dimensional integral.
- For a closed surface, by convention ***n*** is chosen to point out.

Why the orientation of the surface is important:

A circle in the plane:



A circle rotated out of the plane:



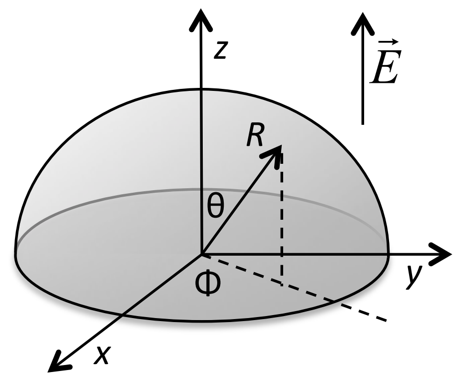
It presents a smaller target to a field that points into the plane.

Flux through a Surface

Example: Flux through a hemisphere:

$z = (R^2 - x^2 - y^2)$. $\boldsymbol{E} = (0, 0, E_z)$.

It's easier to use spherical coordinates, (r, θ, ϕ) because r is constant on the surface.



$$\hat{n} = (\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta) \Rightarrow \vec{E}\cdot\hat{n} = E_z\cos\theta$$

$$dA = R^2\sin\theta\, d\theta\, d\varphi$$

Therefore:

$$\text{Flux} = \int_0^{2\pi}\int_0^{\pi/2} E_z R^2 \sin\theta\cos\theta\, d\theta\, d\varphi$$

$$= E_z R^2 \tfrac{1}{2} 2\pi \qquad \text{using: } \int \sin x\cos x\, dx = \tfrac{1}{2}\sin^2 x. \text{ Look it up!!}$$

$$= E_z(\pi R^2) \qquad \text{As expected, I hope.}$$

End 8.26/13