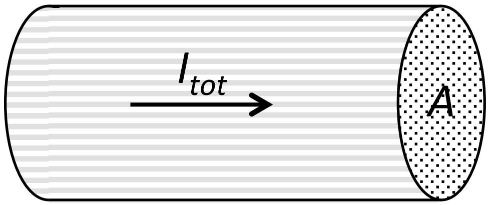
Conductors (2.5)

A conductor is a material in which: $E \neq 0 \Rightarrow j \neq 0$

Remember that electric current is the flow of charge through a surface.
So it makes sense to talk about the amount of flow per unit surface area.
This is the current density, j (Amps per m^2).



If a wire of cross section A carries total current, I_{tot}, then $j = I_{tot}/A$. (assuming the current is uniformly distributed)

In most common materials (*e.g.*, metals) the ***j***-***E*** relationship is nearly linear:

$$\vec{j} = \sigma \vec{E}$$

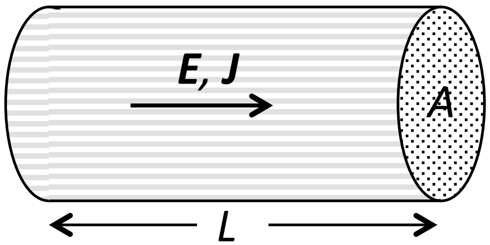
This is a vector equation. The current flows in the direction of ***E***.
Units of σ: (Amp/m^2) / (Volts/m)

σ is often considered to be a property of the material, but be careful:

- It depends on environmental quantities (*e.g.*, temperature).
- It depends on ***E*** (for large ***E***) – that is, the ***j***-***E*** relationship becomes nonlinear.
- ***j*** might not be parallel to ***E***. (anisotropic material).
  In this case, σ is a tensor (3×3 matrix), not a number.

Practical Matters

Resistivity, ρ, is $1/\sigma = E/J$. Its units are Ohm·m. Let's see how this works:



The voltage drop along the wire is $V = EL$.

The total current is $I_{\text{tot}} = JA = \frac{E}{\rho}A = V\left(\frac{A}{\rho L}\right) \equiv \frac{V}{R}$

So, $R \equiv \frac{\rho L}{A}$ is ohms, as expected.

Consider copper: $\rho = 1.7\times10^{-8}\ \Omega\cdot$m.

A 5 m length of 18 gauge wire (diameter = 1.0 mm) has an 0.11 Ω resistance.

This is not large, but is significant is you want to send 10 A through it (1.1 V drop).

Electrostatics

For now, we're considering situations in which the charges aren't moving (***j*** = 0).

Therefore:

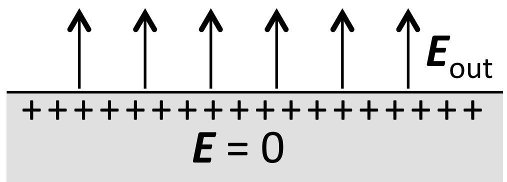
- ***E*** = 0 inside a conductor, to keep ***j*** = 0.
- ρ = 0 inside a conductor, because $\rho = \varepsilon_0 \vec{\nabla} \cdot \vec{E} = 0$.
- All charge resides on the surface of a conductor.
  It can't be anywhere else.
- A conductor is an equipotential: $\Delta V = \int \vec{E} \cdot d\vec{l} = 0$
- At the surface of a conductor, ***E***(outside) must be perpendicular to the surface. (See next slide)

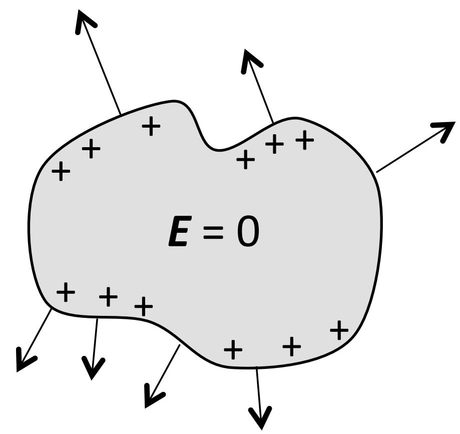
These statements are true for all conductors in the static situation.

They are true for perfect conductors, $\sigma \Rightarrow \infty$, in all situations,
even if there is time dependence.

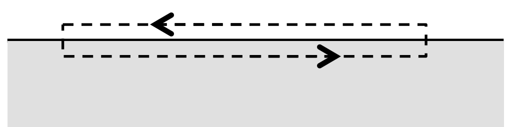
Why $\vec{E} \perp$ Surface of a Conductor

Look at a small patch of the conductor's surface:





Consider $\vec{\nabla} \times \vec{E} = 0$, and do this loop integral:



The contribution from the bottom leg is zero, but the contribution from the top is only zero if ***E*** is perpendicular to the path. (The sides are too short to matter.) So, $\vec{E} \parallel \hat{n}$

Use Gauss' law to determine the magnitude of ***E***. Answer:

$$\vec{E}_{out} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

I'll let you do the work.
Draw a Gaussian surface that is partly inside and partly outside the conductor.

Unfortunate nomenclature:
σ is used for both surface charge density and electrical conductivity.

Another way to express the last property. Use $\vec{E} = -\vec{\nabla} V = -\frac{dV}{dn}$:

$$\vec{E}_{\text{out}} = \frac{\sigma}{\varepsilon_0}\hat{n} \Rightarrow \boxed{\frac{dV}{dn} = -\frac{\sigma}{\varepsilon_0}}$$

Here, *n* (without the hat) means the distance from the surface.

This relation is often a useful boundary condition when solving Poisson's equation.

An important corollary: (2.5.2)

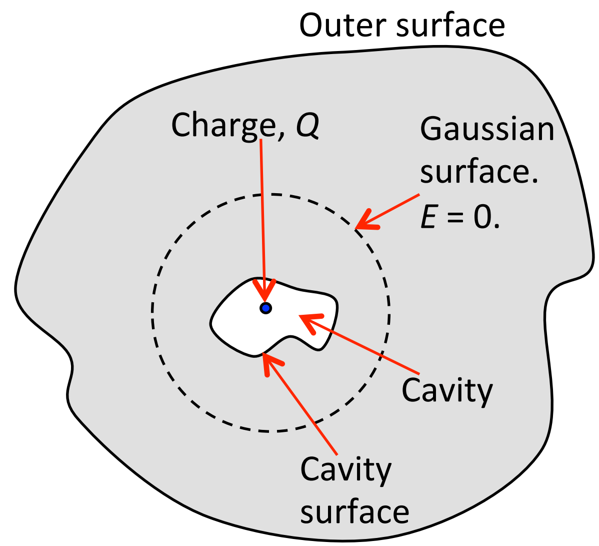
The interior of a conductor is unobservable form the outside.

This means that the field outside a conductor is completely determined by what's happening on its surface. We can't infer anything about its internal structure without actually looking inside.

Note: The *V* boundary condition is usually more useful than the ***E*** one, because scalar equations are usually easier to solve than vector ones.

Let's look inside a conductor:

Suppose, for example, there is a cavity containing some charge. Because it's a cavity, we can draw a Gaussian surface that lies entirely within the conductor. ***E*** = 0 on this surface. Therefore, the cavity surface must have –*Q*, distributed so that the field of *Q* is completely cancelled.



Two conclusions:

- The total charge inside a conductor must be zero, even if the geometry is complicated.
- Any charges inside the conductor cannot affect the charge on the outer surface, because ***E*** = 0 just inside the surface.

The big question: What determines the distribution of charge on the outer surface?

I claim: There is only one way to distribute this charge to satisfy the boundary conditions. We'll prove uniqueness theorems soon (not right now).

End 9/6/13