First exam: Wednesday, October 2, in class

- The first exam will cover through Griffiths, section 3.3 (separation of variables).
- This was completed at the beginning of last Monday's lecture.
- Homework 5 (due today) covers this material as well.
  The solution will be posted on Tuesday.
- Monday will be a review lecture.

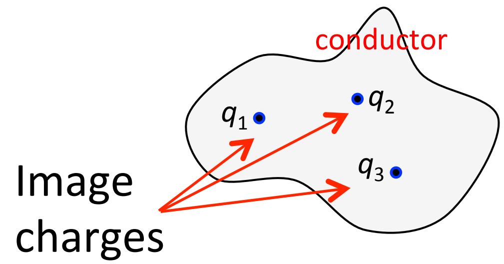
Suggested practice homework problems from Griffiths:

- Chapter 1: Vector review:
  1.30-1.34, 1.38-1.40, 1.53-1.58
- Chapter 2: E field:
  2.2, 2.4-2.8, 2.10-2.11, 2.14, 2.16-2.17, 2.20, 2.22-2.24, 2.31, 2.33-2.36, 2.41, 2.45
- Chapter 3: Laplace's Equation:
  3.3, 3.6, 3.8, 3.13, 3.17, 3.18, 3.20, 3.22, 3.24, 3.25

These problems cover the concepts, but remember that homework problems are usually too long to be good exam questions.

Interlude

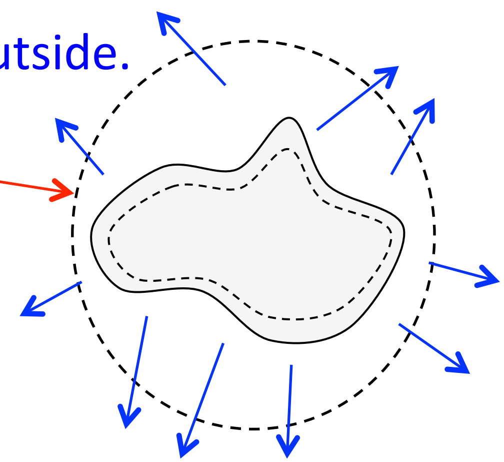
The homework grader tells me that there was a lot of confusion about problem 1 of homework 4:



Show that the total actual surface charge equals the sum of the image charges.

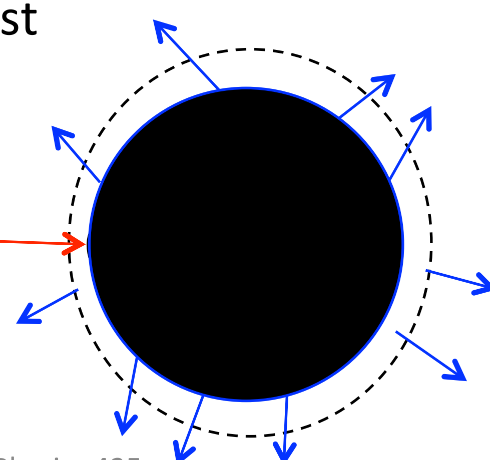
Apparently, confusion often arose from the failure to remember that:

Image charges do not give the correct electric field inside the conductor's outer surface, but they do give the correct field outside.



The Gaussian surface outside the conductor, has the same electric flux from the image charge as from the actual charge. Therefore, the total enclosed charge must be the same in both cases.

Which one is it, the conductor, or an insulator with embedded charge?
They are different on the inside, but not outside.

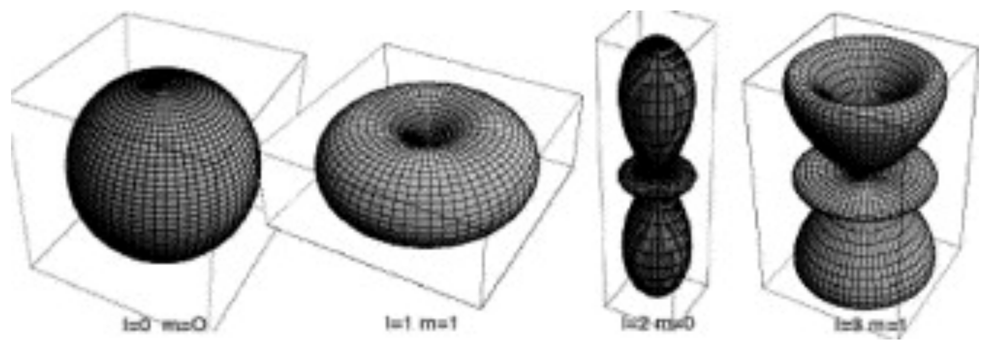


Polarization (4.1)

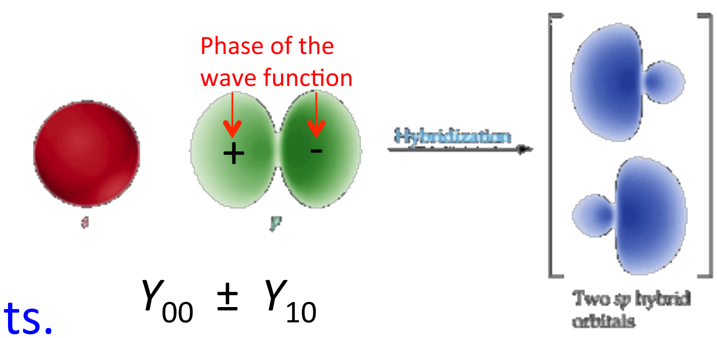
Remember Physics 214? (I thought not …)

The ground state (or any $\ell = 0$ state) wave function of the electron is proportional to the $Y_{00}(\theta,\phi)$ spherical harmonic ($= 1/\sqrt{4\pi}$). Because it's spherically symmetric, atoms in this state have no electric dipole moment.

In fact, all of the Y_{lm} probability distributions have enough symmetry that $\boldsymbol{p} = 0$ when the atom is in one of those states.



However, superpositions of the $Y_{\ell m}$ do not, in general, give symmetric charge distributions.

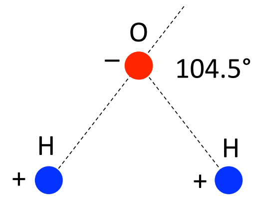


$Y_{00} \pm Y_{10}$

Atoms in these states have electric dipole moments.

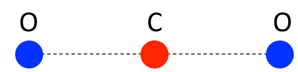
Some molecules naturally have dipole moments.

Consider H_2O:



Oxygen has a stronger affinity for electrons than hydrogen does, so it tends to be negatively charged, as shown. Also, the bonds are not lined up.
➔ H_2O has a large dipole moment. This is an important property of water.

Some molecules (*e.g.*, CO_2) don't have an intrinsic dipole moment, due to symmetry:



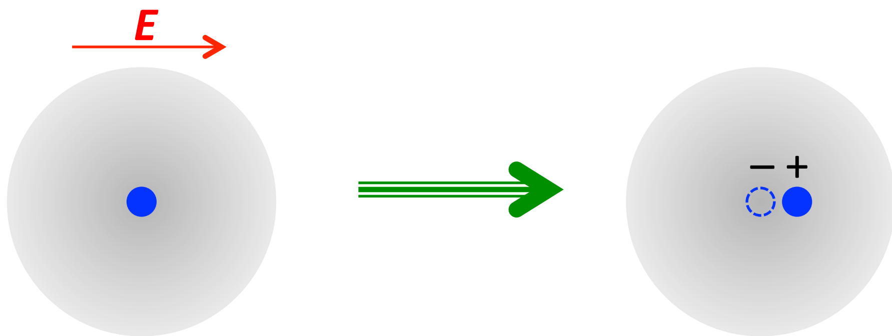
However, most materials will attain $\boldsymbol{p} \neq 0$ if placed in an external $\boldsymbol{E}$ field, because the electrons can move around. This is called polarizability.

We will study what happens when we have bulk material, made up of large numbers (10^{24}) of polarizable constituents.

Atomic Polarizability

(Molecules, too)

If you put an atom in an electric field, the electrons and nucleus feel opposite forces.

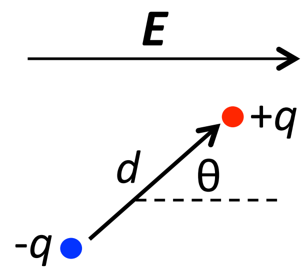


The atom gains a dipole moment, $\vec{p} = \alpha \vec{E}$ (for small ***E***).
We can, in principle, calculate *α*, but it's messy and requires QM.
I'll just treat it as an empirical property of the material.

We will calculate the force and torque exerted by an applied *E* field on an electric dipole, in order to understand the behavior of materials in an electrical environment.

Forces on dipoles:

The electric field is doing double duty. It polarizes the atom, then it exerts a force and torque on the resulting dipole. For now we'll take the dipole as given (represented by ±*q* in the figure.



Case 1: Uniform E field

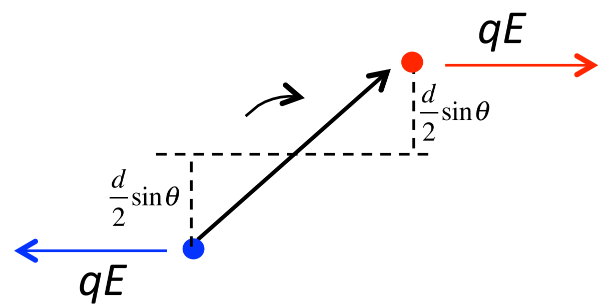
The force on the two charges cancels, so ***F*** = 0.

However, there is a net torque:

$$N = 2\left(\tfrac{d}{2}\sin\theta\right)qE = pE\sin\theta$$

In vector form: $\vec{N} = \vec{p}\times\vec{E}$

Here, $\vec{N}$ points into the screen.



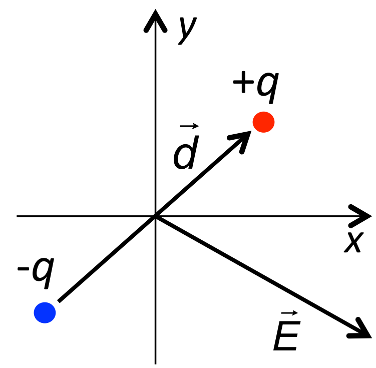
Electric dipoles tend to align with ***E***.

Case 2: Nonuniform E field

The forces no longer cancel.

$$F_x = q\left(E_{x+} - E_{x-}\right) = q\left(\frac{\partial E_x}{\partial x}\Delta x + \frac{\partial E_x}{\partial y}\Delta y + \frac{\partial E_x}{\partial z}\Delta z\right)$$

where $\left(\Delta x, \Delta y, \Delta z\right) = \vec{d}$, the displacement from $-q$ to $+q$.



I have deliberately chosen ***p*** and ***E*** not to align with the coordinate axes.

So (using $q\vec{d} = \vec{p}$): $F_x = \left(\vec{p}\cdot\vec{\nabla}\right)E_x$

It looks like a dot product.

$$\equiv p_x\frac{\partial}{\partial x} + p_y\frac{\partial}{\partial y} + p_z\frac{\partial}{\partial z}$$

E_y and E_z are similar, so we have:

$$\vec{F} = \left(\vec{p}\cdot\vec{\nabla}\right)\vec{E}$$

This looks weird, but it's just a compact way to write the derivatives.

Example:

A speck of dust near a charged surface (*e.g.*, furniture).
The dust is polarized along ***E***: $\vec{p} = \alpha\vec{E}$ Call that the *x*-direction.

$$\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E} \Rightarrow F_x = \alpha E_x \frac{\partial E_x}{\partial x} = \frac{\alpha}{2}\frac{\partial E_x^2}{\partial x} = \frac{\alpha}{2}\frac{\partial E^2}{\partial x}$$

← Because E_y and E_z are zero.

It feels a force in the direction of increasing magnitude of the field.
It is attracted to the furniture and sticks to it.

Notes:

If the E field is curving, so that $\frac{\partial E_{y,z}}{\partial x} \neq 0$, then there will be sideways forces, but they tend to be very small. The field doesn't curve much over the size of a dust speck.

There is a similar effect with magnets. A piece of magnetically polarizable material is attracted to a region of stronger field. (Think "refrigerator magnet")

The SI units for polarizability, α, are (C·m)/(V/m) = (C·m^2)/V.
The polarizability of a hydrogen atom is 7.4×10^{-41} (C·m^2)/V. (Does that make sense?)

How much is a hydrogen atom distorted by an electric field?

Suppose $|\boldsymbol{E}| = 10^7$ V/m (a moderately strong field).
Then $p = 7.4\times10^{-34}$ C·m.
e = 1.6×10^{-19} C, so the displacement between electron and proton is 4.6×10^{-15} m.
That's only a little bigger than the size of the nucleus, so the distortion is small.
That's the reason that atomic polarization effects tend to be linear.

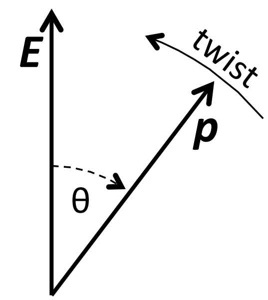
How big are everyday electric fields?

For example, when you slide across the rug and spark your sibling, the electric field needed to produce the spark is $|\boldsymbol{E}| \sim 3\times10^6$ V/m. This is called the "dielectric strength" of air. (The field strength at which one can get sparks).

Potential Energy:

When there are forces and torques, there is potential energy.
Remember that torques do work:

$$W = \int_{\theta_{\text{ref}}}^{\theta} N\,d\theta = -pE \int_{\theta_{\text{ref}}}^{\theta} \sin\theta'\,d\theta' = +pE\left(\cos\theta - \cos\theta_{\text{ref}}\right)$$



So, pick θ = π/2: $U = -W = -\vec{p}\cdot\vec{E}$

The stable orientation is to align ***p*** with ***E***.
Remember, the head of ***p*** is the + charge.

End 9/27/13