Can we write *A* for an infinite, straight wire?

Assume the wire is along the z axis, and the current is along $+\hat{z}$.

Use cylindrical coordinates.

- ***B*** is in the $\hat{\varphi}$ direction and is a function only of *s*.
- ***A*** can have either (or both): $A_s(z)$ or $A_z(s)$.

Let's see if we can get away with $A_z(s)$ only:

$$B_\varphi = \frac{\mu_0 I}{2\pi s} = -\frac{\partial A_z}{\partial s} \quad \Rightarrow \quad A_z(s) = \frac{\mu_0 I}{2\pi}\ln(s) + C$$

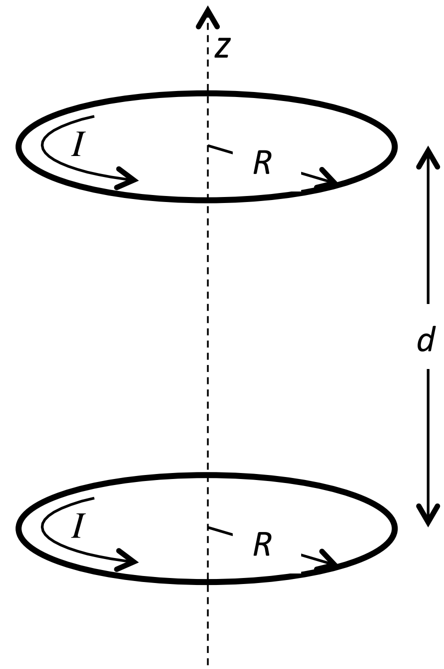
We can make this a bit prettier by defining $C \equiv -\frac{\mu_o I}{2\pi}\ln(s_0)$.

Then: $\vec{A}(s) = \frac{\mu_0 I}{2\pi}\ln\left(\frac{s}{s_0}\right)\hat{z}$. Note that A blows up as $s \to \infty$.

Can you find a solution using only A_s? Does it satisfy $\vec{\nabla}\cdot\vec{A} = 0$?

Practical example (not a vector potential problem):

The Helmholtz coil is a very simple way to make a nearly uniform ***B*** field over a reasonably large volume.



Consider two coaxial circular loops of radius R, each carrying current I. They are at $z = \pm d/2$.

Let's calculate B_z at $z = 0$ on the z-axis.

Here is the formula for a single loop at $z = 0$:

$$B_z(z) = \frac{\mu_o I}{2} \frac{R^2}{\left(R^2 + z^2\right)^{3/2}}$$

So, the on-axis field is:

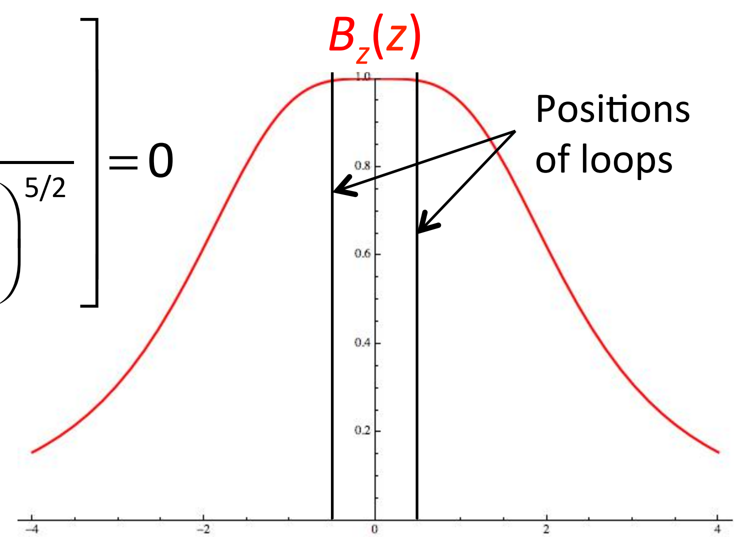
$$B_z(z) = \frac{\mu_o I R^2}{2} \left[\frac{1}{\left(R^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}} + \frac{1}{\left(R^2 + \left(\frac{d}{2} - z\right)^2\right)^{3/2}} \right]$$

At z = 0: $B_z(0) = \dfrac{\mu_o I R^2}{\left(R^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}}$. $B_x = B_y = 0$ (by symmetry).

How uniform is *B*?

- By symmetry, the first derivative should be zero.

$$\left.\frac{dB_z}{dz}\right|_{z=0} = \frac{3\mu_o I R^2}{2}\left[\frac{-\frac{d}{2}}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{5/2}} + \frac{+\frac{d}{2}}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{5/2}}\right] = 0$$



- The second derivative is:

$$\left.\frac{d^2B_z}{dz^2}\right|_{z=0} = \frac{3\mu_o I R^2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{7/2}}\left(d^2 - R^2\right)$$

What do you think $\frac{d^3B_z}{dz^3}$ is?

For completeness, you should take the derivatives of B_s also.

This also equals zero if we make $d = R$.

Helmholtz coils are very cheap, and one has easy access to the uniform field region, unlike with a solenoid.

Magnetic Fields in Matter (Ch. 6)

We want to describe magnetic fields in the presence of polarizable materials, in a manner similar to the electric case. We have free currents that we can manipulate, and the material will generate bound currents in response.

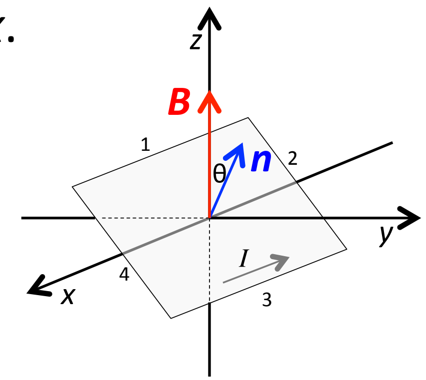
The magnetic situation is much more complicated than the electric one, for two reasons:

- The magnetic susceptibility, χ_m, can be positive (paramagnetism) or negative (diamagnetism).
- Many materials have a nonlinear response. In fact, ferromagnetic materials have non-zero polarization even in the absence of an applied ***B***. (Ferroelectrics are uncommon.)

Recall that polarization is the dipole moment density, so let's describe magnetic dipoles.

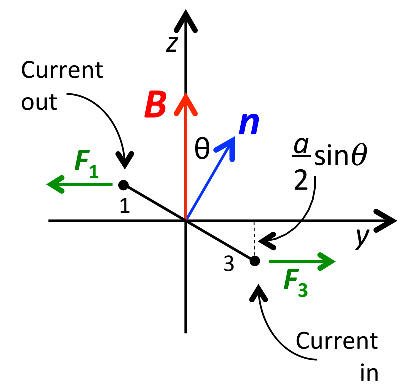
Calculate the torque on a small (square) current loop in a *B* field.

Let ***B*** be along *z*, but give the loop an arbitrary rotation about *x*.



Sides 1 and 3 are parallel to the *x* axis.
Sides 2 and 4 are rotated by θ out of the *x*-*y* plane.
That is, ***n*** is in the *y*-*z*-plane, rotated by θ away from *z*.
The current is I.

The net force is zero, but the torque is not, because there is a lever arm on sides 1 and 3. Look at it along the *x*-axis:



$F_1 = F_3 = Iba$,

So the torque is $2\times(a\sin\theta/2)\times(IBa) = \underbrace{(Ia^2)}_{\text{magnetic moment}}B\sin\theta$.

We can write this in vector form (as with electric dipoles):

$\vec{m} = IA\hat{n}$ $\quad$ A is the area of the loop.

$\vec{N} = \vec{m}\times\vec{B}$ $\quad$ In this example, $\vec{N} \parallel \hat{x}$.

End 10/21/13