The radial force has changed: $$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2} + ev'B_z = m_e\frac{v'^2}{R}$$

$$\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}}_{= m_e \frac{v^2}{R}}$$

v' is the new speed

So, $$ev'B = \frac{m_e}{R}\left(v'^2 - v^2\right) \approx \frac{m_e}{R}\left(2v'\Delta v\right)$$

For small Δv

Thus: $$\Delta v \approx \frac{eRB_z}{2m_e}$$

So that: $$\Delta\vec{m} = -\frac{1}{2}e\Delta vR\hat{z} = -\frac{e^2R^2}{4m_e}B_z\hat{z}$$

$\Delta\boldsymbol{m}$ always opposes $\boldsymbol{B}$. This is called diamagnetism.

The effect does not depend on the direction of orbital motion or on the sign of the electron charge.

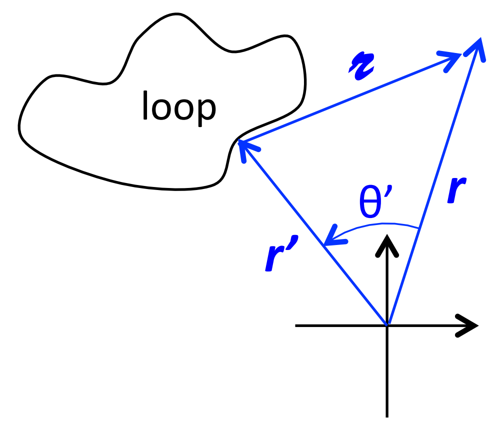
In order to describe magnetic materials, we want to use the multipole expansion in a similar way to what we did in electrostatics.

Start with: $\vec{A} = \dfrac{\mu_0 I}{4\pi} \oint \dfrac{d\vec{l}}{\mathscr{r}}$ (for a current loop)

As before, write: $\dfrac{1}{\mathscr{r}} = \dfrac{1}{r} \sum_{\ell=0}^{\infty} \left(\dfrac{r'}{r} \right)^{\ell} P_{\ell}(\cos\theta')$ (equation 3.94)

So, $\vec{A} = \dfrac{\mu_0 I}{4\pi} \left[\dfrac{1}{r} \oint d\vec{\ell}' + \dfrac{1}{r^2} \oint r' \cos\theta' d\vec{\ell}' + \ldots \right.$

Monopole always = 0 — dipole — Quadrupole, *etc.*



We can write the dipole term this way:

$$\vec{A}_{\text{dipole}} = \frac{\mu_0 I}{4\pi r^2} \oint \left(\hat{r} \cdot \vec{r}\,' \right) d\vec{\ell}'$$

Now, use a vector identity (see G, equation 1.108)

$$\oint (\hat{r}\cdot\vec{r}\,')\,d\vec{\ell}\,' = -\hat{r}\times\vec{a},\ \text{where } \vec{a}\equiv\oint \vec{r}\,'\times d\vec{\ell}\,'$$

a is called the vector area of the loop.
It is defined for all loops, including non-planar ones.
$\vec{a} = A\hat{n}$ for a planar loop.

You have HW to study vector areas.

With this identity and definition, $\vec{A}_{\text{dipole}} = \dfrac{\mu_0}{4\pi r^2}\vec{m}\times\hat{r}$, where $\vec{m}\equiv I\vec{a}$.

Magnetized Objects (6.2.1)

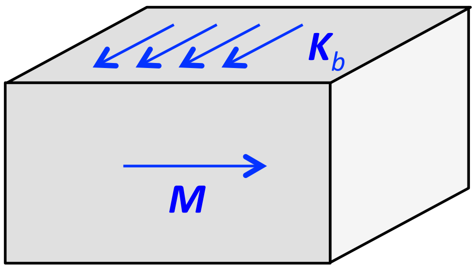
A magnetized object is described by a magnetic dipole density, ***M***.
In that case, we must integrate to calculate ***A***:

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{Vol'} \frac{\vec{M}(\vec{r}\,')\times\vec{\mathscr{r}}}{\mathscr{r}^2}\, dVol'$$

M is the magnetic analog of ***P***.

We could calculate ***B*** directly, but the math is much simpler if we use ***A***.

We are going to describe the source, $\frac{\vec{M}\times\vec{\mathscr{r}}}{\mathscr{r}^2}$, as being due to bound currents. The physics is similar to electric polarization being due to bound charges. As before, I'll present an intuitive description. G does the rigorous derivation.

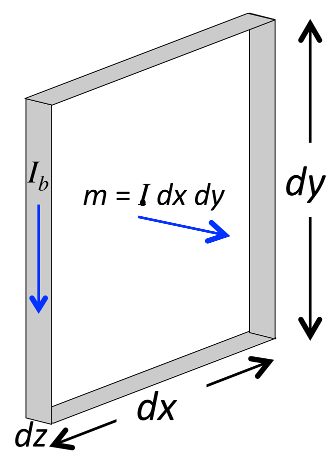


Consider a volume that is full of magnetic dipoles (all pointing the same direction). We will think of them as little current loops:

The density of dipoles is $\rho_d = \dfrac{1}{dx\,dy\,dz}$.

Therefore, the magnetic moment density (per volume) is

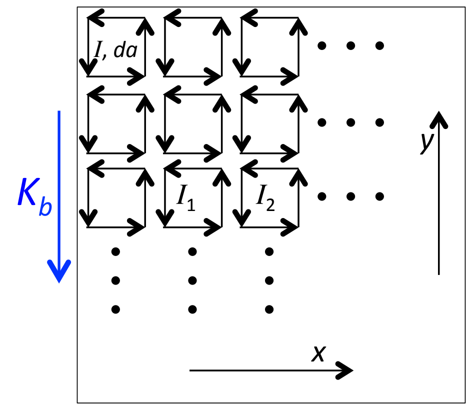
$$\vec{M} = \rho_d \vec{m} = \frac{I_b}{dz}\hat{z}$$



Look at the end view.

If the dipole density is constant, then all the internal currents cancel, and we are left with a current density around the boundary: $K_b = I_b/dz = M$.

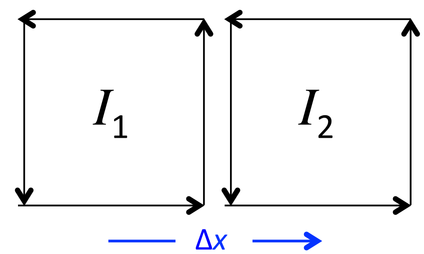
End view



Actually, this is a cross product. ***M*** points along +*z*, and ***n*** points along -*x* (on the left surface in the picture), so $\mathbf{K}_b = \mathbf{M}\times\mathbf{n}$. There is no surface current on the end surface, where ***M*** is parallel to ***n***. If the surface is tilted, the sinθ dependence of the cross product gives the correct answer.

If the dipole density is not constant, then the internal currents don't cancel. The resulting volume current density is:

$$J_b = \left(\frac{I_2 - I_1}{\Delta x}\right)\left(\frac{1}{dz}\right) = \frac{dM}{dx}$$



This is also a cross product (actually a curl): $\vec{J}_b = \vec{\nabla}\times\vec{M}$

So, as with bound charge, there are two kinds of bound current:

- A surface current, $\mathbf{K}_b$, due to the abrupt change of ***M*** at the surface.
- A volume current, $\mathbf{J}_b$, due to non-uniform magnetization.

End 10/25/13