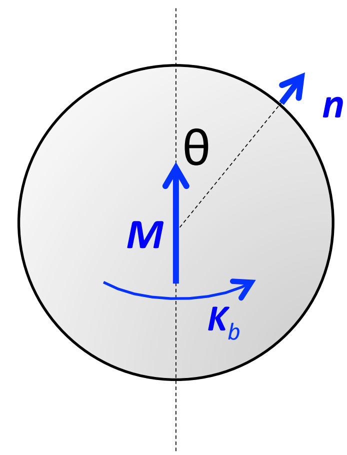
Examples:

Uniformly magnetized sphere. (G, ex. 6.1, p. 264)



$$\vec{J}_b = \vec{\nabla} \times \vec{M} = 0$$

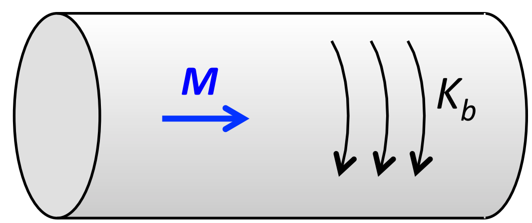
$$\vec{K}_b = \vec{M} \times \hat{n} = M \sin\theta \, \hat{\varphi}$$ On the surface.

This is the same ***K*** and ***B*** as is produced by a rotating shell of surface charge. (See G, ex 5.11, p. 236.)

$$\vec{B}_{\text{in}} = \tfrac{2}{3}\mu_0 \vec{M}$$ uniform

$$\vec{B}_{\text{out}} = \text{ dipole field: } \vec{M}_{\text{tot}} = \tfrac{4}{3}\pi R^3 \vec{M}$$

Uniformly magnetized rod.



$$\vec{J}_b = \vec{\nabla} \times \vec{M} = 0$$

$$\vec{K}_b = \vec{M} \hat{\varphi}$$

This is exactly like a solenoid.

The Field due to Free Currents (6.3.1)

Just as it was useful to define D such that $\vec{\nabla}\cdot\vec{D}=\rho_f$,

it is useful (even more so) to define H such that $\vec{\nabla}\times\vec{H}=\vec{J}_f$. ← ***H*** is produced by free currents

Use Ampere's law: $\frac{1}{\mu_0}\vec{\nabla}\times\vec{B}=\vec{J}=\vec{J}_f+\vec{J}_b=\vec{\nabla}\times\vec{H}+\vec{\nabla}\times\vec{M}$

So, $$\vec{H}=\frac{1}{\mu_0}\vec{B}-\vec{M}$$

In the vacuum ($\boldsymbol{M}$ = 0), $\boldsymbol{B} = \mu_0\boldsymbol{H}$, similar to $\boldsymbol{D} = \varepsilon_0\boldsymbol{E}$.

H is more useful than D, because we have more control over $\boldsymbol{J}_f$ than ρ_f.

Nomenclature and units: (They're confusing to me …)

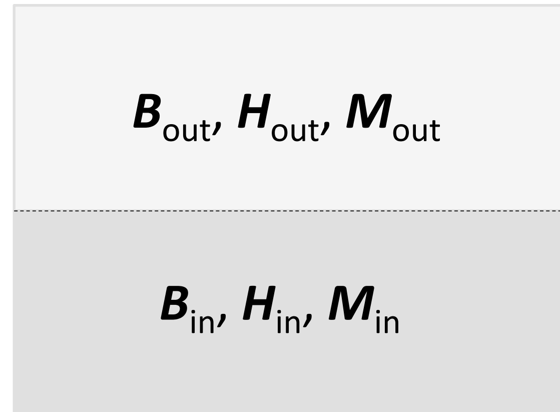
- ***B*** is often called the magnetic flux density.
- ***H*** is usually called the magnetic field strength.
- ***M*** is called the magnetization.

  Units of ***H*** and ***M*** are amperes/meter.

Solving Magnetic Problems

We have boundary conditions at surfaces:

1) $\vec{\nabla}\times\vec{H}=0$ at the surface, so: $H_{\parallel}(\text{in})=H_{\parallel}(\text{out})$



2) $\vec{\nabla}\cdot\vec{B}=0$ always, so: $B_{\perp}(\text{in})=B_{\perp}(\text{out})$

Also notice that

$$\vec{\nabla}\cdot\vec{H}+\vec{\nabla}\cdot\vec{M}=0 \quad\Rightarrow\quad H_{\perp}(\text{in})-H_{\perp}(\text{out})=-\left(M_{\perp}(\text{in})-M_{\perp}(\text{out})\right)$$

$\vec{\nabla}\cdot\vec{H}\neq 0$ in general.

3) $\vec{\nabla}\times\vec{B}=\mu_0\vec{J} \quad\Rightarrow\quad B_{\parallel\text{out}}-B_{\parallel\text{in}}=\mu_0\left(\vec{K}\times\hat{n}\right)$

Total surface current: bound + free

In order to go farther, we must have a relationship between ***B*** or ***H***, and ***M***.

We will assume a linear response: $\vec{M}=\chi_m\vec{H}$

Why do we use $\vec{M} = \chi_m \vec{H}$ instead of $\chi_m \vec{B}$?

We make magnetic fields by controlling free currents, which are the source of ***H*** (not ***B***).

In the electric situation, we make electric fields by controlling voltage (total charge). Hence, ***E***, not ***D***.

Some more nomenclature:

We have: $\vec{B} = \mu_o \left(\vec{H} + \vec{M}\right) = \underbrace{\mu_0 \left(1 + \chi_m\right)}_{\equiv \mu} \vec{H} = \mu \vec{H}$

and: $\vec{M} = \chi_m \vec{H}$

χ_m is called the magnetic susceptibility (like $\chi_{e)}$.
μ is called permeability (unlike ε, permittivity).

Typical values of χ_m are ~ $\pm 10^{-5}$ (small).
Magnetic effects are small in most materials, except ferromagnets, which we'll discuss later.

End 10/28/13