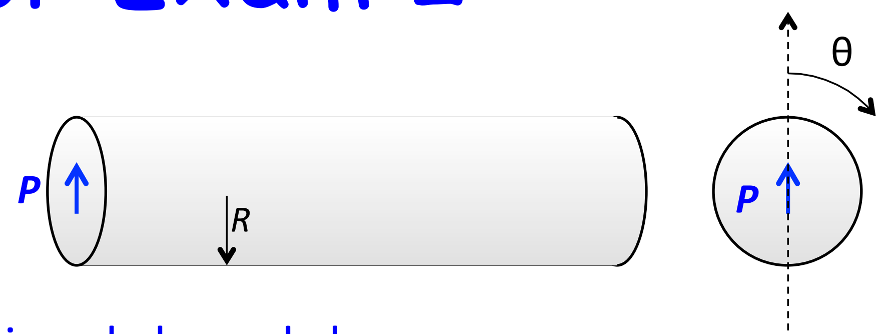
Review for Exam 2

1) A very long cylinder is polarized perpendicular to its axis, as shown. Calculate E.



E is determine by the total charge. There is only bound charge:

Volume: $\rho_b = -\vec{\nabla}\cdot\vec{P} = 0$. Surface: $\sigma_b = \vec{P}\cdot\hat{n} = P\cos\theta$.

The potential can only contain cosθ terms.

Inside: $V_{in}(s,\theta) = V_i\, s \cos\theta$

Outside: $V_{out}(s,\theta) = V_o\, s^{-1} \cos\theta$

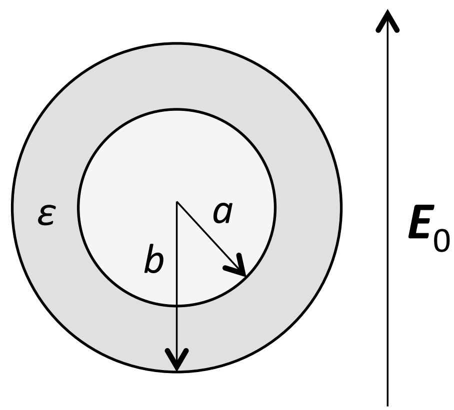
Boundary conditions:

- Continuity of V at $s = R$: $V_i = V_o R^{-2}$
- Gauss's law at $s = R$: $-V_i - V_o R^{-2} = P/\varepsilon_0$

$V_i = -P/2\varepsilon_0$. $V_{in}(s,\theta) = -(P/2\varepsilon_0)\, s \cos\theta$ ← Uniform field along ***P***.

$V_o = -PR^2/2\varepsilon_0$. $V_{out}(s,\theta) = -(PR^2/2\varepsilon_0) \cos\theta/s$

2) A conducting sphere (radius = *a*) is surrounded by a dielectric coating (outer radius = *b*). It is placed in an initially uniform field, $\boldsymbol{E}_0$. Write the equations that describe the boundary conditions on $V(r,\theta)$.



This is similar to the dielectric sphere example that we did in lecture, except that there is now a second surface (at r = *a*) that must be an equipotential. Note that, as before, only $\ell = 1$ will survive.

The dielectric region is now allowed to have both *r* and r^{-2} dependence, so write:

$V_{in}(r,\theta) = (A_1 r + B_1 r^{-2}) \cos\theta$

At r = ∞: $\boldsymbol{E}(r,\theta) = \boldsymbol{E}_0$ $\quad V(r,\theta) = -E_0\, r \cos\theta$

$V_{out}(r,\theta) = (C_1 r^{-2} - E_0 r) \cos\theta$

At $r = b$: V_{in} = Vout: $\quad (A_1 b + B_1 b^{-2}) = C_1 b^{-2} - E_0 b$ ←

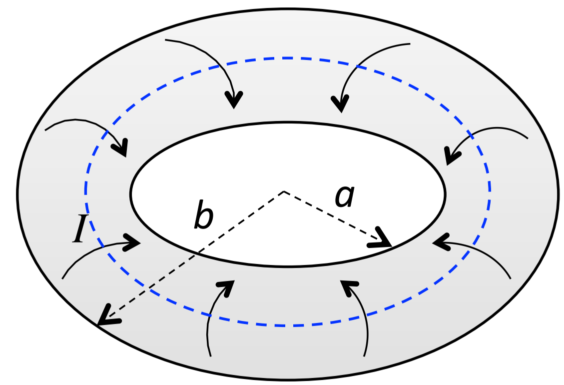
$$\varepsilon \frac{\partial V_{in}}{\partial r} = \varepsilon_o \frac{\partial V_{out}}{\partial r} \qquad \varepsilon(A_1 - 2B_1 b^{-3}) = \varepsilon_0(-2C_1 b^{-3} - E_0)$$ ←

At $r = a$: V_{in} = constant $\quad A_1 a + B_1/a^2 = 0$ ←

That gives us three equations for A_1, B_1, and C_1.

3) Toroidal field.

This "donut" shaped surface has N turns of wire wrapped around it, carrying current I. Calculate $\boldsymbol{B}$ inside the donut.



This problem has cylindrical symmetry.

Draw an Amperian loop (the blue dashed line) of radius r ($a < r < b$) inside the donut. The total current flowing through this loop is NI. Ampere's law tells us that B = $\mu_0 I/2\pi r$. It circulates clockwise in the picture.

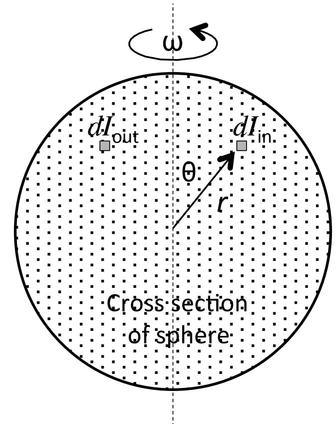
Every loop that lies outside the donut has no flux, so $\boldsymbol{B} = 0$ outside.

Also note that the result does not depend on the shape of the donut, as long as there is cylindrical symmetry (so that $\int \vec{B} \cdot d\vec{\ell} = 2\pi r B$).

Toroidal magnets are used in magnetic confinement fusion research.

3) A uniformly charged (density ρ throughout the volume)sphere of radius R, spins with angular speed ω. Calculate the magnetic dipole moment of the sphere.

The sphere can be thought of as a collection of current loops of radius r and position θ, as shown. Each loop contributes magnetic moment:



$dm = AdI = \pi(r\sin\theta)^2 dI = \pi\rho\omega\, r^4 \sin^3\theta\, dr d\theta$

$dI=(\rho\omega r\sin\theta)\, r dr d\theta$

Integrating: $m_{\text{tot}} = (4/15)(\pi\rho\omega)R^5 = (1/5)Q_{\text{tot}}\omega R^2$

4) A uniformly magnetized rod with square cross section ($a\times a$) is bent into a circle with inner radius R. Calculate the bound currents, $\boldsymbol{J}_b$ and $\boldsymbol{K}_b$ and the total current.

$\boldsymbol{M}$ points in the ϕ direction and is not a function of s, ϕ, or z, so:

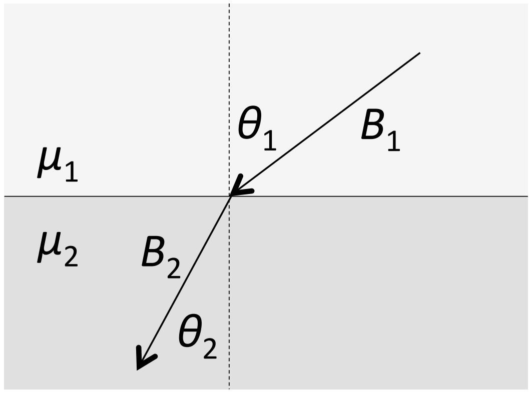
$\vec{J}_b = \vec{\nabla}\times\vec{M} = \frac{M}{s}\hat{z}.$

$\vec{K}_b = \vec{M}\times\hat{n} = +M\hat{z}$ at $s = R$, $-M\hat{z}$ at $s = R + a$, and $\pm\hat{s}$ on the top and bottom.

The total volume current is $2\pi Ma^2\hat{z}$.

The total surface current is $2\pi RaM\hat{z} - 2\pi(R+a)aM\hat{z}$, so the total current is zero.

5) The magnetic field will bend when it crosses the boundary between two magnetic materials of different permeability. What is the relationship between θ_1 and θ_2?



H tangential is continuous, so $\frac{B_{1\|}}{\mu_1} = \frac{B_{2\|}}{\mu_2}$.

B normal is continuous: $B_{1\perp} = B_{2\perp}$

Take the ratio: $\frac{B_{1\|}}{\mu_1 B_{1\perp}} = \frac{B_{2\|}}{\mu_2 B_{2\perp}} \Rightarrow \frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$