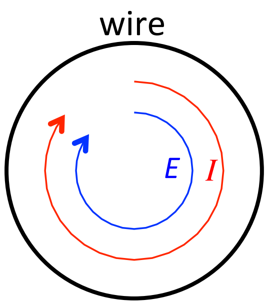
Electrodynamics (Chapter 7)

We'll begin to deal with time dependence.

The first important change: $\vec{\nabla}\times\vec{E}\neq 0$ ← Faraday discovered this.

This means, of course, that $\oint \vec{E}\cdot d\vec{l}\neq 0$



Can we use this to drive current around a loop?
The answer is yes, but not in a static situation.

I'll assume Ohm's Law: $\boldsymbol{J} = \sigma\boldsymbol{E}$.
σ is called the conductivity of the material. $\rho = 1/\sigma$ is called resistivity.

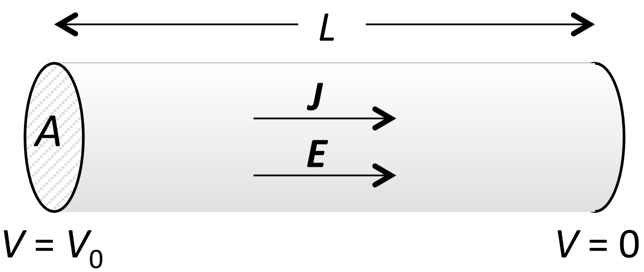
Most materials are either conductors: $\rho \sim 10^{-7}$ (in SI units – V·m/A)
or insulators: $\rho \sim 10^{+10}$.
A few are semiconductors: $\rho \sim 10^{0}$ to10^{3} (strongly temperature dependent).

For the time being, I'll ignore magnetic fields and forces (due to the currents).

Resistivity and resistance

Resistivity, ρ, is a property of the material. (We'll usually use conductivity.)
Resistance, R, also depends on the size and shape of a particular piece of material.

Consider this example (p. 286):
Assume $\mathbf{E}$ is uniform. A good approximation.



$\mathbf{J} = \sigma\mathbf{E}$, so $I = JA = \sigma EA$.

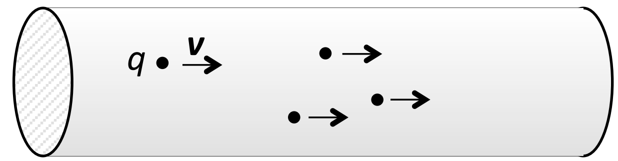
Define resistance: $$R \equiv \frac{V}{I}$$

Here, $R = \frac{V_0}{I} = \frac{EL}{\sigma EA} = \frac{L}{\sigma A}$. It depends on A, L, and σ, but not on E (due to linearity).
The shape of A doesn't matter, as long as it is constant along the length.

Units of R: $\frac{\text{m}}{\frac{\text{Amp}}{\text{V}\cdot\text{m}}\text{m}^2} = \frac{\text{Volt}}{\text{Amp}}$, as expected ($\equiv$ Ohm)

Note that for steady currents, $\vec{\nabla}\cdot\vec{J} = \frac{\partial\rho}{\partial t} = 0 \Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{1}{\sigma}\vec{\nabla}\cdot\vec{J} = 0$
if σ doesn't depend on position.
No charge inside a conductor.

Power needed to maintain a steady current.



For each particle: $\text{Power} = Fv = \frac{F}{q}qv = Eqv$

Of the particle density is ρ_p, then:

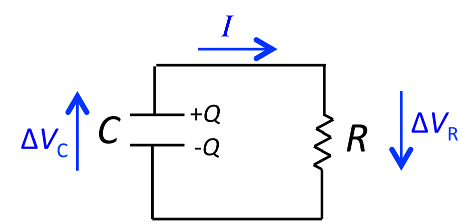
Per unit volume: $\text{Power} = E(q\rho_p v) = EJ$

For the whole object: $\text{Power} = (EJ)A_{\text{rea}}L = \dfrac{J^2}{\sigma}AL = I^2\dfrac{L}{\sigma A} = I^2 R$

Current loops: (A P212 review)

1) A simple RC circuit:

Suppose that at $t = 0$, the capacitor is charged to some potential $V_0 = Q_0/C$. Calculate the current, $I(t)$, in the circuit and the voltage, $V(t)$ on the capacitor.



Use $\int \vec{E}\cdot d\vec{l} = 0$. Assuming that the wire is a perfect conductor ($E = 0$), this reduces to two terms: ΔV_C and ΔV_R: $\Delta V_C + \Delta V_R = 0$. $Q/C - IR = 0$.

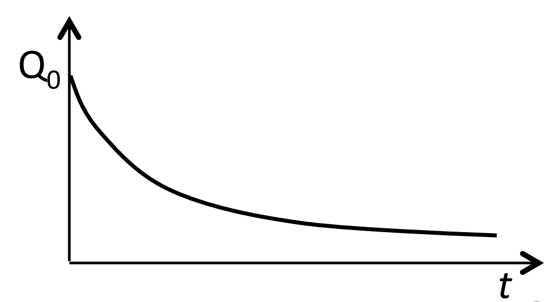
Kirkhoff's first law

We can solve this using $I = -\frac{dQ}{dt}$.

$$\frac{1}{Q}\frac{dQ}{dt} = -\frac{1}{RC} \Rightarrow Q(t) = Q_0 e^{-\frac{t}{RC}}$$

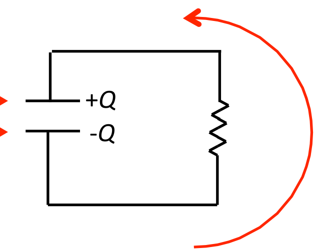
Exponential decay:

RC is the "time constant"

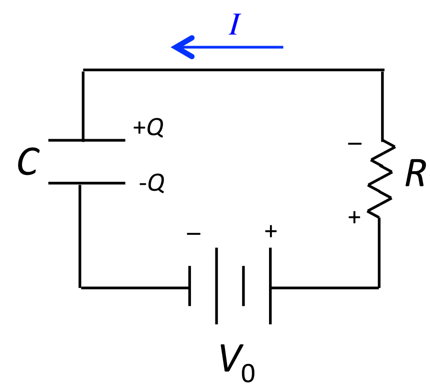


How the electronic move:

No big deal.



2) Now, put a battery in the circuit.
Suppose the capacitor is initially uncharged.
$Q(t=0) = Q_0 = 0$



Kirkhoff's Law is: $V_0 - IR - \frac{Q}{C} = 0$

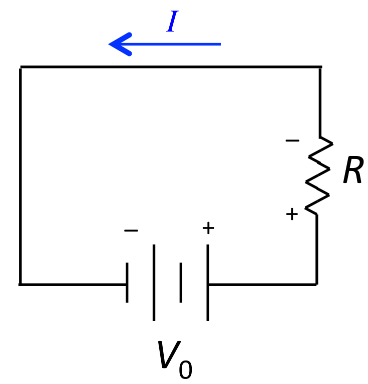
Or: $CV_0 - Q - RC\frac{dQ}{dt} = 0$

Solution: $Q(t) = CV_0 + (Q_0 - CV_0)e^{-\frac{t}{RC}}$ (Q_0 =0 here)

Again, no big deal. The capacitor charges up until $Q = CV_0$, at which point, there is no voltage agross the resistor (*i.e.*, $I = 0$).

Now, consider a simpler circuit …

3) Only a battery and a resistor.
$I = V_0/R$, a constant current.



The electrons go round and round forever (or until the battery is dead).

How is that possible?
In particular, the electrons are going the wrong way through the battery!

There are non-electrostatic forces acting inside the battery that move the electrons against the electrostatic force. These forces supply the I^2R power that is deposited in the resistor.

Note: A battery is not a capacitor.
For example, a car battery is capable of delivering 300 Amp-hours (1.1×10^6 C).

A 12 V 'battery" would have to have 9×10^4 F.

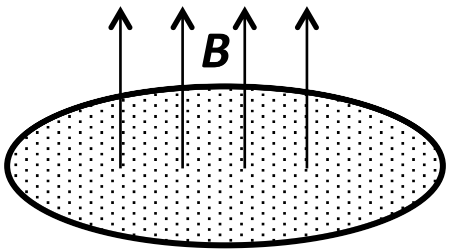
Other Circuit Components:

We won't talk about transistors and diodes, but we will talk about inductors.

To understand inductors, we need to learn about Faraday's Law, discovered by Faraday (duh!) and others in 1830-1.

In a nutshell: $\vec{\nabla} \times \vec{E} \neq 0$ when there is time variation.

Faraday discovered it this way:
Consider a loop of wire in a magnetic field.



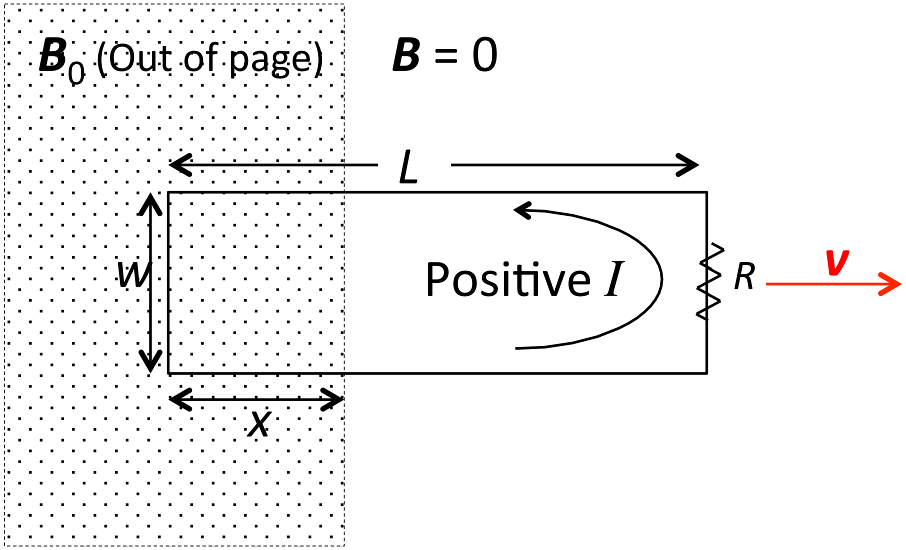
He found (empirically) that: $$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot \hat{n}\, dA \equiv -\frac{d\Phi}{dt}$$

The loop integral of E equals minus the rate of change of the magnetic flux.

Example: (G, p. 295)

At the instant shown:

$$\Phi = B_0 wx \text{ and } \frac{d\Phi}{dt} = -B_0 wv$$



So, $\oint \vec{E} \cdot d\vec{l} = +B_0 wv$

Positive is in the CCW direction.

$$I = B_0 wv/R$$

Power is going into the resistor. Where is the energy coming from?

We must pull on the wire to cancel the magnetic force on the left-hand segment: $F = IB_0 w$. The power is $P = Fv = IB_0 wv = I^2 R$.

We need a non-electrostatic source of energy.

This is how electric generators work.

End 11/8/13