Exam2, Problem 1

The answers to problem 1 on the exam made me realize that several conceptual issues need to be addressed.

- There are two mechanisms for producing polarization in a material.
  - Response of a permeable material to an applies external field. We assume that the response is linear: $\vec{M} = \chi_m \vec{H}$.
  - Intrinsic polarization, ***M***, even in the absence of an applied field. This is clearly not a linear response: χ_m doesn't have any meaning.

  The slab in problem 1 was the latter type. One must calculate ***B*** and ***H*** from ***M***.
- $\vec{B} = \mu_0(\vec{H} + \vec{M})$ is always true. It follows from the definition of ***H***. One corollary, which follows from $\vec{\nabla} \cdot \vec{B} = 0$, is that $\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M}$.
- There is no free current in this problem. This implies that $\vec{\nabla} \times \vec{H} = \vec{J}_f = 0$. However, this does *not* imply that ***H*** = 0! In order for this to be true we also need $\vec{\nabla} \cdot \vec{M} = 0$. In fact, when $\vec{\nabla} \times \vec{H} = 0$, the solution for ***H*** is mathematically identical to an electrostatics problem, with $-\vec{\nabla} \cdot \vec{M}$ playing the role of the charge density (*i.e.*, it is the source for the ***H*** field).

Other issues that many students had:

- Be careful when interpreting Amperean loops. $\oint \vec{B} \cdot d\vec{l} = 0$ does not necessarily imply ***B*** = 0, unless you have other information about B (*e.g.*, which direction it is pointing). In this problem, ***B*** is uniform (and non-zero in part a) inside the slab. The loop integral will always be zero for a uniform field.
- ***B*** responds to all currents, whether bound or free, volume or surface. $\boldsymbol{J}_b = 0$ does not imply that ***B*** = 0. You need to know $\boldsymbol{K}_b$.
- Many people didn't apply the right-hand rule correctly to the fact that the current in the top surface points opposite to that in the bottom surface. This implies that the fields add between the surfaces, and cancel outside.
- This problem has ±*z* symmetry (or antisymmetry). Several students wrote asymmetrical answers, which can't possibly be correct.

End 11/11/13