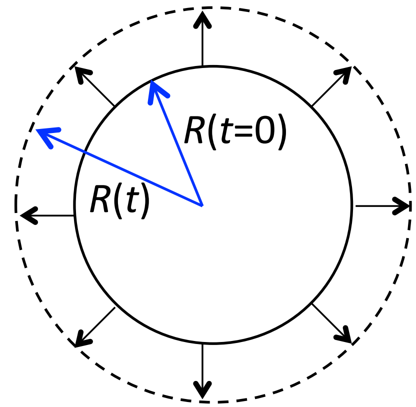
One does not need a non-uniform or changing ***B***, as long as the flux changes.

Consider a circular loop with a time dependent radius:

At any given time, $\frac{d\Phi}{dt} = B_0 2\pi R \frac{dR}{dt}$, so (if R is increasing)



$\boldsymbol{B}_0$ (out of picture)

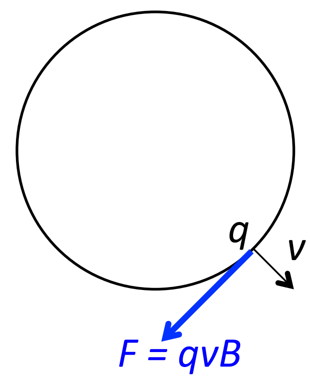
There is a clockwise EMF (electromotive force), and current will flow.

Beware:

Motional EMF is simply a result of the Lorentz force:

However:

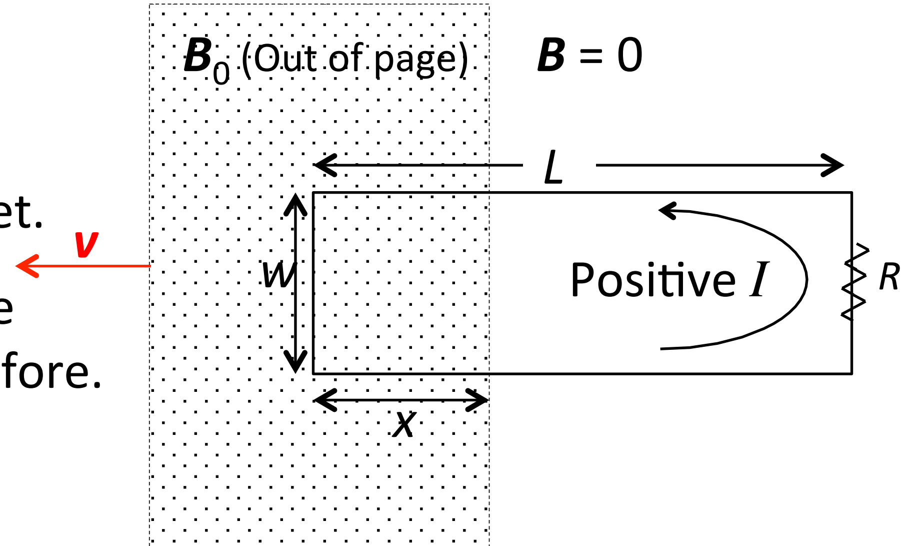
The situation is much more subtle than this.



Faraday's Law (7.2.1)

Consider two variations on last lecture's rectangular loop example.

1) Don't move the loop - move the magnet.



By the equivalence of inertial reference frames, this is the same situation as before. However, it is not motional EMF in this frame, because the wire is not moving.

What is pushing the electrons around the loop?

There must be an electric field in the wire's reference frame!

E must point along w (but only in the $\boldsymbol{B}_0$ region). Let's calculate E:

$Ew = IR = B_0wv$. In vector form: $\vec{E} = \vec{v} \times \vec{B}_0$.

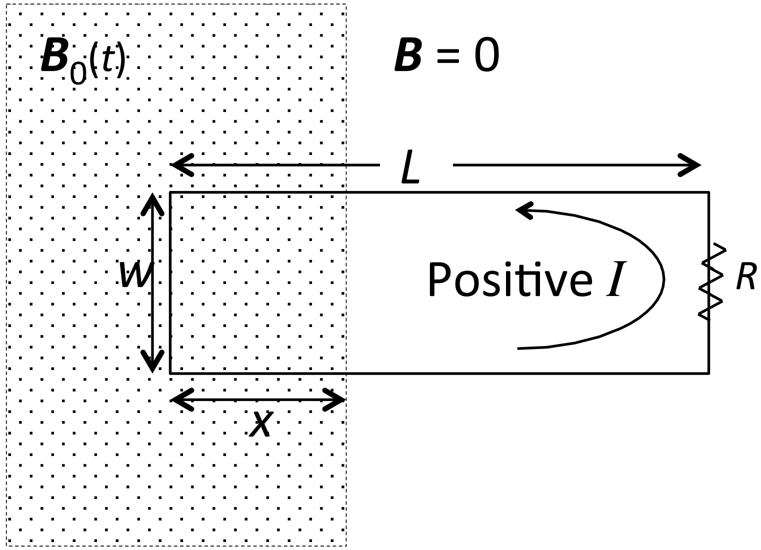
This is the ***E*** field seen by the wire, which is moving to the right with respect to the reference frame in which there is only a magnetic field.

The moral: E (and B) have different values in different (moving) reference frames.

This is the beginning of special relativity, which we'll study next semester.

2) Don't move the loop or the magnet.
Let $\boldsymbol{B}_0$ be a function of time. Then:

$$\frac{d\Phi}{dt} = wx\frac{dB_0}{dt} \neq 0$$



Do we get a current (an EMF)?
Yes! (an empirical result).

This is not motional EMF in any reference frame.

So, the rule: $\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi}{dt}$

is not simply $\vec{F} = q\vec{v} \times \vec{B}$, although they are consistent.

We will learn next semester how they are connected.

Differential form of Faraday's law:

This relationship:

$$\oint_{\text{loop}} \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int_{\text{surface}} \vec{B}\cdot\hat{n}\,dA$$

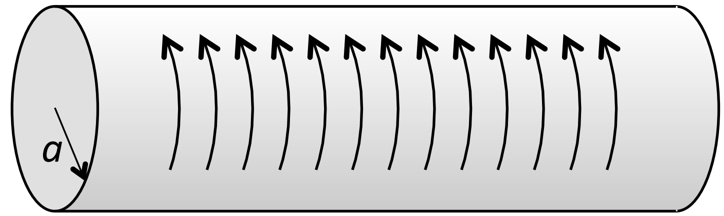
$$= \int_{\text{surface}} \left(\vec{\nabla}\times\vec{E}\right)\cdot\hat{n}\,dA$$

Holds for any loop and any spanning surface.
Therefore (as with Gauss's Law), the integrands must be equal:

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

Example: (Illustrating a subtlety)

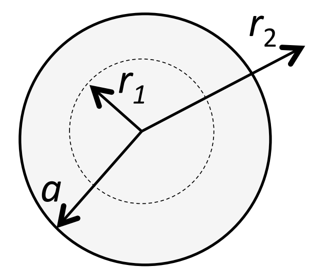
A long solenoid. Radius = a.
Current = I_{wire}. n turns per unit length.



Let's calculate ***E*** inside and outside, if
I is varying – $I(t)$.

$$\vec{B}(t) = \mu_0 n I(t)\hat{z} \quad \text{inside}$$
$$= 0 \quad \text{outside}$$

By symmetry, ***E*** can only depend on s (no ϕ or z dependence).
Draw the Amperean loop shown (radius r_1 or r_2).



Inside: $2\pi r_1 E_\phi = -\mu_0 n \dot{I}(\pi r_1^2) \Rightarrow E_\phi = -\frac{\mu_0 n r_1}{2}\dot{I}$

Outside: $2\pi r_2 E_\phi = -\mu_0 n \dot{I}(\pi a^2) \Rightarrow E_\phi = -\frac{\mu_0 n a^2}{2 r_2}\dot{I}$

But wait !!!

B = 0 everywhere outside.
How can a varying B inside the solenoid affect ***E*** somewhere else?

Responses:

a) $\vec{\nabla} \times \vec{E} = 0$ outside You've done this problem before.
So, at least we're consistent with Faraday's law.

But, that doesn't address the problem.

b) The real answer is that our equations don't (yet) properly take into account propagation of the ***E*** and ***B*** fields (*i.e.*, electromagnetic waves).
This is a P436 topic.

For now, we'll restrict ourselves to local effects.

Note that when $\vec{\nabla} \times \vec{E} \neq 0$, we can't define a potential, $V(\mathbf{r})$ such that $\vec{E} = \vec{\nabla} V$.

However, we can still use Kirkhoff's loop law when analyzing a circuit, by including the induced EMF as another voltage source (like a battery).

End 11/13/13