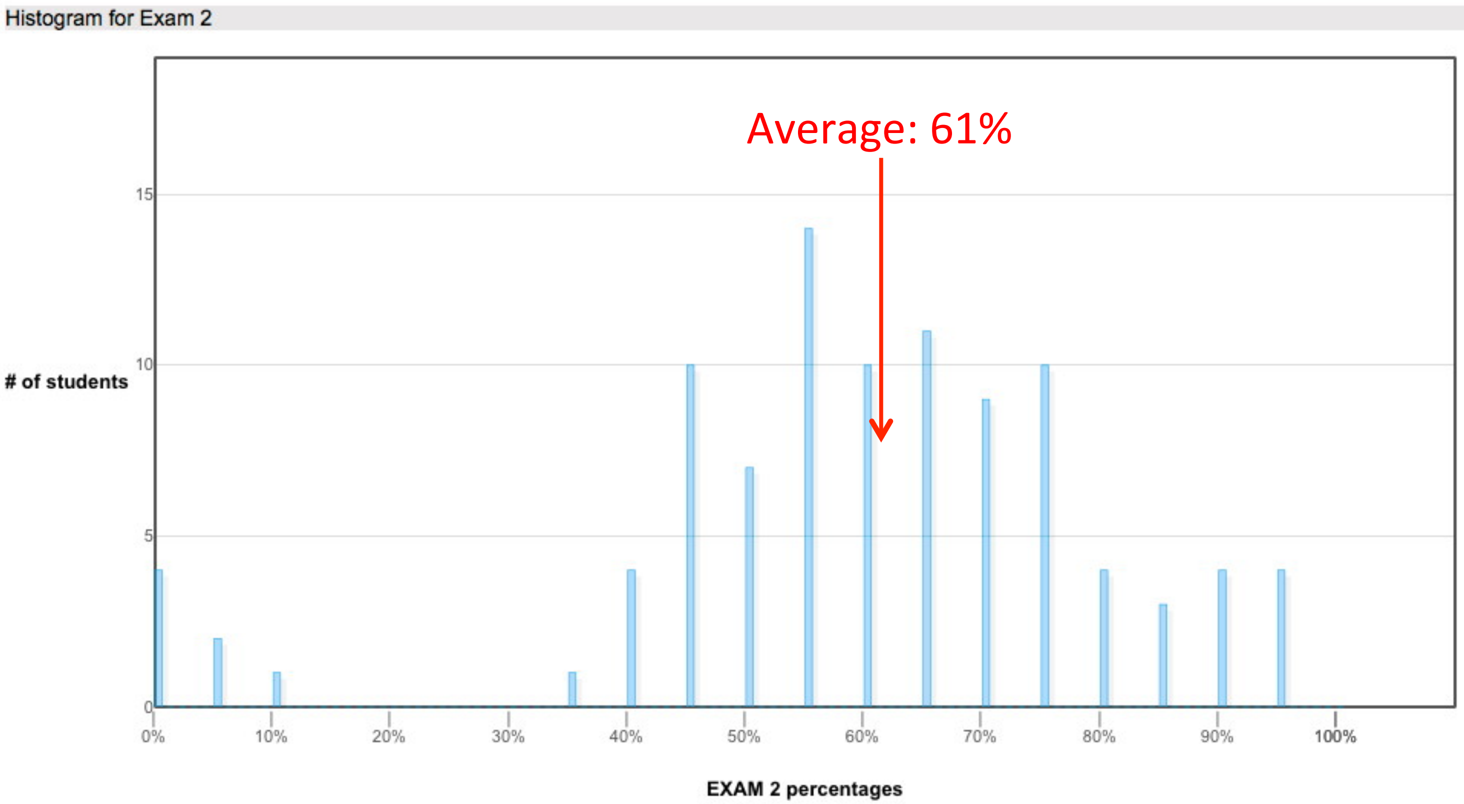
Exam 2 results



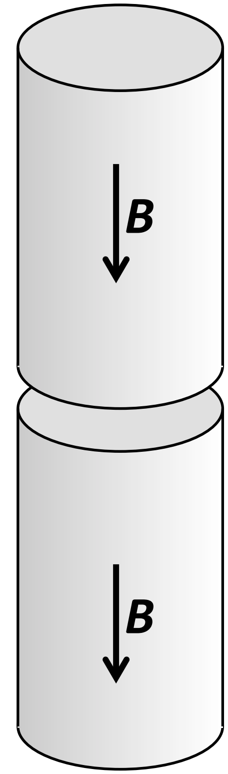
The most common conceptual issues:

- See the Nov. 11 lecture.

Example: The betatron: (G, problem 7.48)

This particle accelerator was invented at UIUC by Donald Kerst in 1940. It is a direct application of Faraday's Law.

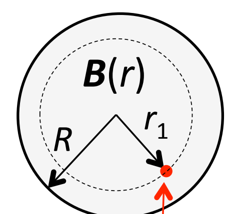
Consider an electromagnet with circular pole tips and a small gap. Put an electron at $r = r_1$ and begin to turn on the ***B*** field.



There will be both electric and magnetic forces.

E will have only a ϕ component, so the electron will begin to accelerate tangentially. $F = qE = -\frac{q}{2\pi r1}\frac{d\Phi}{dt}$

We want the electron to move in a circle of constant r_1. Can we manage this?



B fields cause circular motion (because ***F*** is perpendicular to ***v***)

$$qvB = m\frac{v^2}{r} \Rightarrow qBr = mv$$

We want r to remain unchanged as ***B*** and ***v*** increase. (***v*** increases due to ***E***.)

Constant r_1 means that: $$qBr_1 = mv \Rightarrow qr_1 \tfrac{dB}{dt} = m\tfrac{dv}{dt} = qE$$

So, we need: $$E(r_1) = r_1 \frac{dB(r_1)}{dt}$$

These are the fields at r_1, where the electron is.

E is determined by Faraday's Law: $$2\pi r_1 E = -\frac{d\Phi}{dt} \Rightarrow -\frac{1}{2\pi r_1}\frac{d\Phi}{dt} = r_1 \frac{dB}{dt}$$

Integrate from $t = 0$ to the time of interest: $$B(r_1) = -\frac{1}{2\pi r_1^2}\Phi$$

This means that we want the field at r_1 to be half of the average field for $r < r_1$.

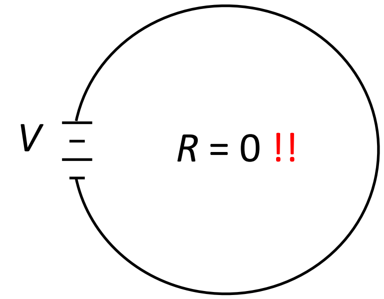
The analytical solution that works for all values of r_1 is $B(r)$ = const/r , but this diverges at $r = 0$. In practice, one must pick a preferred r_1 and shape the B field to work at that radius.

Parts of Kerst's original apparatus are on display outside the lecture room.

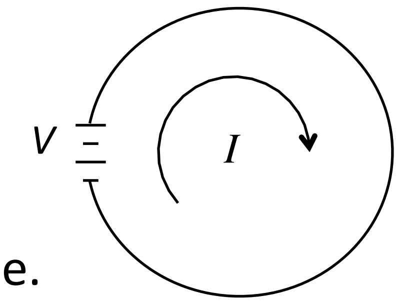
Lenz's Law

Lenz's Law is a result of the minus sign in Faraday's law.

Let's look at a wire that has no resistance. Attach a battery to it: What will happen?



Current will flow, but not an infinite current. $V \neq IR$ here.
Let's see why. Suppose that at some instant there is current, I:



It will produce a ***B*** field through the loop, into the picture and proportional to I. I am interested in the magnetic flux.

Unfortunately, ***B*** is not uniform, so this flux is not easy to calculate.
Let's call the proportionality constant L: $\Phi = LI$

Can we determine the time dependence, $I(t)$, of the current?
Faraday's law provides the answer.

The battery will cause the current to increase. What is $\frac{dI}{dt}$?

Faraday's Law tells us the EMF: $\mathcal{E} = -\frac{dI}{dt}$

The circuit equation (Kirkhoff's loop law) is thus: $V + \mathcal{E} = 0$

The minus sign is Lenz's Law. It tells us that the induced EMF opposes the driving voltage.

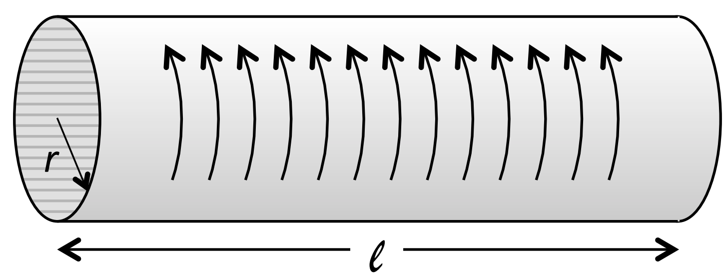
$$V - L\tfrac{dI}{dt} = 0 \quad \Rightarrow \quad I(t) = I_0 + \tfrac{V}{L}t$$

As a result of Lenz's Law, the current in a circuit cannot increase arbitrarily rapidly.

The proportionality constant, *L*, is called the self inductance of the loop. It depends on the geometry and also on the presence (if any) of magnetic materials, which affect the flux.

Example: an air core solenoid (no iron)

$B = \mu_0 K$. $\Phi = (\pi r^2) N B = (\pi r^2)(n\ell)(\mu_0 n I) = LI$

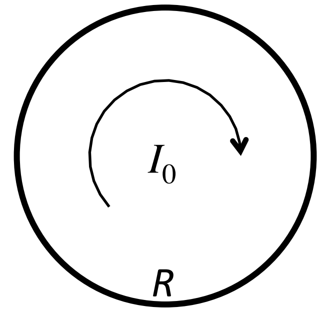


If we fill the solenoid with magnetic material, B increases (μ_0 becomes μ), and so does *L*.

Some numbers: $\mu = 10^3\,\mu_0$ $\ell = 3$ cm
$n = 10^4$/m $r = 5$ mm $L = 3\times10^{-2}$ Henry (= Volts/(Amp/sec))

Example: LR circuit

Suppose there is no driving force, but there is an initial current, I_0, and the loop has resistance, R.



Then, $-IR - L\frac{dI}{dt} = 0 \Rightarrow I(t) = I_0 e^{-\frac{R}{L}t}$ $\frac{L}{R}$ is the time constant

The current decays away, and dumps energy into the resistor:

$$E_{\text{tot}} = \int_0^\infty I^2 R\,dt = I_0^2 R \int_0^\infty e^{-\frac{2Rt}{L}}\,dt = \tfrac{1}{2} I_0^2 L$$

We'll worry soon about where this energy is coming from.

Note:

Without the minus sign in Lenz's Law, the current would spontaneously increase.

End 11/15/13