What about the current ratio?

$$L_p \frac{dI_p}{dt} + M\omega \frac{L_s}{MR} V_{\text{in}} \sin(\omega t) = V_{\text{in}} \cos(\omega t)$$

Using the solution for I_s on the last page.

$$\frac{dI_p}{dt} = \frac{V_{\text{in}}}{L_p}\left(\cos(\omega t) - \frac{\omega L_s}{R} \sin(\omega t) \right)$$

$$I_p(t) = \frac{V_{\text{in}}}{L_p}\left(\frac{1}{\omega} \sin(\omega t) + \frac{L_s}{R} \cos(\omega t) \right)$$

$$= V_{\text{in}}\left(\frac{1}{\omega L_p} \sin(\omega t) + \frac{L_s}{L_p R} \cos(\omega t) \right)$$

For large L_s ($\omega L_s >> R$), the sin() term is small,

and $\frac{I_s}{I_p} = -\frac{L_p}{M} = \frac{N_p}{N_s}$ ⇐ The current ratio.

Let's check energy conservation.

$$\langle \text{Power in} \rangle = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} V_{\text{in}}(t) I_p(t)\,dt$$

$$= \frac{\omega}{2\pi} V_{\text{in}}^2 \int_0^{\frac{2\pi}{\omega}} \cos(\omega t)\left[\frac{1}{\omega L_p}\sin(\omega t) + \frac{L_s}{L_p R}\cos(\omega t)\right]dt = \frac{1}{2} V_{\text{in}}^2 \frac{L_s}{L_p R}$$

$$\langle \text{Power out} \rangle = \frac{\omega R}{2\pi}\int_0^{\frac{2\pi}{\omega}} I_s^2(t)\,dt$$

$$= \frac{1}{2} R \frac{L_s^2 V_{\text{in}}^2}{M^2 R^2} = \frac{1}{2} V_{\text{in}}^2 \frac{L_s}{L_p R}$$ The same.

This leads me back to the question I asked last lecture:

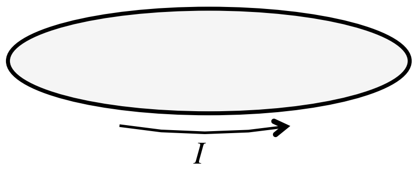
- You saw in last week's discussion that you have to do work $W = \frac{1}{2} L I^2$ to get current flowing in an inductor.
- We saw last lecture that an inductor can deposit energy $E = \frac{1}{2} L I^2$ in a resistor as the current drops to zero.

Where does the energy go between these two events? We want energy to be conserved at all times, so it must be somewhere.

Magnetic Field Energy (7.2.4)

Consider a current loop: $LI = \Phi = \int_{\text{surface}} \vec{B}\cdot d\vec{a}$

$$= \int_{\text{surface}} (\vec{\nabla}\times\vec{A})\cdot d\vec{a} = \oint_{\text{loop}} \vec{A}\cdot d\vec{\ell}$$



So: $W = \frac{1}{2}LI^2 = \frac{1}{2} I \oint_{\text{loop}} \vec{A}\cdot d\vec{\ell} = \frac{1}{2}\oint_{\text{loop}} (\vec{A}\cdot\vec{I})\,d\ell$

If it's a volume current: $W = \frac{1}{2}\int_{Vol} \vec{A}\cdot\vec{J}\, dVol$

Write this in terms of ***B***: $\vec{J} = \frac{1}{\mu_0}\vec{\nabla}\times\vec{B}$

$$W = \frac{1}{2\mu_0}\int_{Vol} \vec{A}\cdot(\vec{\nabla}\times\vec{B})dVol = \frac{1}{2\mu_0}\int_{Vol}\left[\underbrace{\vec{\nabla}\cdot(\vec{A}\times\vec{B})}_{=0 \text{ if fields vanish at } \infty} + \vec{B}\cdot\underbrace{(\vec{\nabla}\times\vec{A})}_{=B}\right]dVol$$

Using math identity #6

$$\boxed{W = \frac{1}{2\mu_0}\int B^2\, dVol}$$

Compare with

$$\boxed{W = \frac{\varepsilon_0}{2}\int E^2\, dVol}$$

See G, p. 192

The energy is stored in the fields!

Comment:

This derivation looks like mere bookkeeping. However, when we discuss E-M radiation next semester, we'll see that field energy (electric and magnetic) has real physical significance.

End 11/20/13