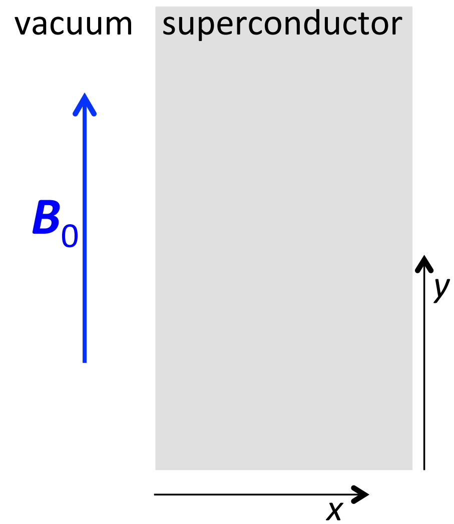
Meissner effect:

Consider this situation:



In the superconductor:

$$\vec{\nabla}\times\vec{J} = -\frac{q^2 n}{m}\vec{\nabla}\times\vec{A} = -\frac{q^2 n}{m}\vec{B}$$ London's condition

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = \vec{\nabla}\left(\underbrace{\nabla\cdot\vec{B}}_{=0}\right) - \nabla^2\vec{B}$$ Math identity

$$= \mu_0\left(\vec{\nabla}\times\vec{J}\right)$$ Maxwell

$$= -\frac{\mu_0 q^2 n}{m}\vec{B}$$ London's condition

So: $\nabla^2\vec{B} = +k\vec{B}$ $\qquad k = \frac{\mu_0 q^2 n}{m}$

Specifically: $\nabla^2 B_y = +kB_y$

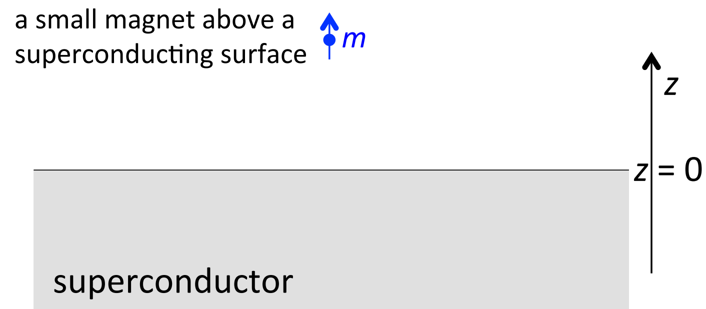
$$B_y = B_0 e^{\pm\sqrt{k}x}$$

($B_{||}$ is continuous)

$\frac{1}{\sqrt{k}}$ is called the penetration depth, λ_L.

λ_L is typically a few tens of nanometers.

Magnetic levitation above a superconductor:

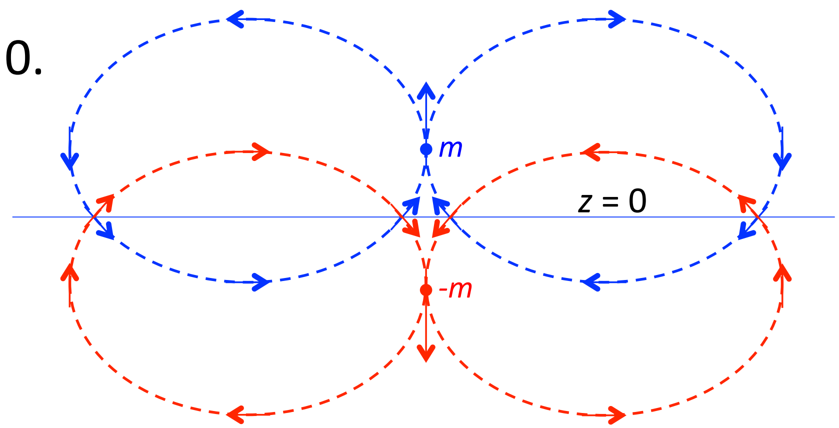


B = 0 inside the superconductor.
Boundary condition: $B_z = 0$ at $z = 0$.

This problem can be solved using images.
Remember the dipole field (Sep. 25 lecture).

$$\vec{B}_{\text{dipole}} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

An image dipole, $-$***m*,** at $-z$ will cancel B_z at $z = 0$.
This works for any orientation of ***m***, not only the one I drew.
The force on ***m*** is the force due to the image.



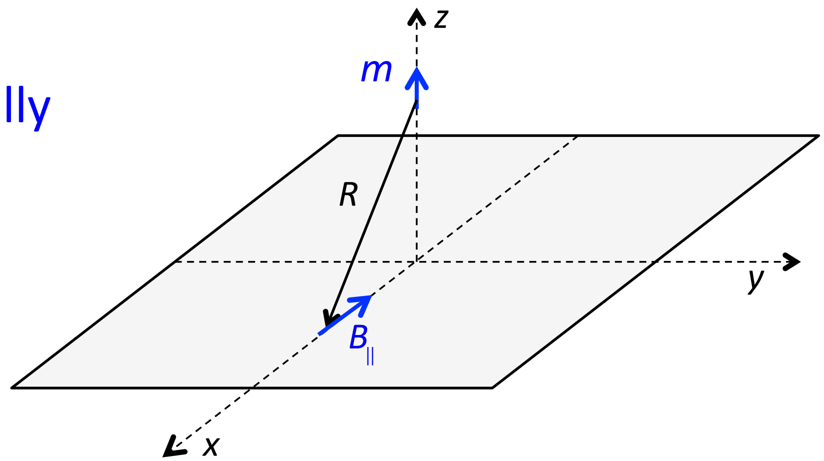
$$\vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}\right)$$

where ***B*** is the field at ***m*** due to the image.
In our situation: $r = 2z$, and $\theta = \pi$ (180°), because the image points down:

$$B_z(\text{due to image}) = -\frac{\mu_0 m}{2\pi r^3} \Rightarrow F_z = \frac{d}{dr}\left(-\frac{\mu_0 m^2}{2\pi r^3}\right) = +\frac{3\mu_0 m^2}{2\pi r^4}$$

The magnet is levitated.

The equilibrium height is given by $\frac{3\mu_0 m^2}{2\pi(2z)^4} = Mg$.



As with image charge, the image dipole is actually surface current. We can calculate this current using the tangential component of ***B*** and Ampere's law. There is cylindrical symmetry, so $B_\parallel$ must be radial (*i.e.*, along *x* in the figure).

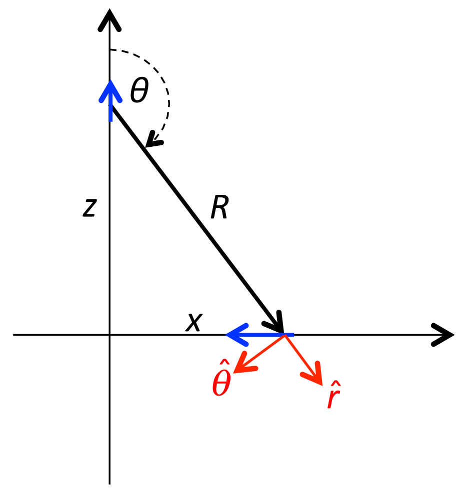
$$B_x = 2\left(B_r \sin\theta + B_\theta \cos\theta\right)$$

$$= \frac{\mu_0 m}{2\pi R^3}\left(2\cos\theta\sin\theta + \sin\theta\cos\theta\right)$$

$$= -\frac{3\mu_0 mzx}{2\pi R^5}\hat{x}$$

$$\sin\theta = \frac{x}{R}$$

$$\cos\theta = -\frac{z}{R}$$



The surface current flows in circles (replace *x* with polar coordinate, *r*):

$$\vec{K} = \frac{1}{\mu_0}\hat{z}\times\vec{B} = \frac{-3mz}{2\pi R^5} r\hat{\varphi}$$

End 12/2/13