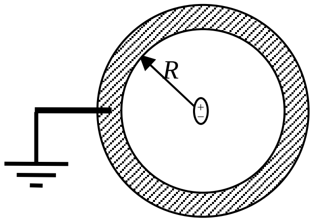
Additional Midterm 2 Review Problems



1. An ideal dipole lies in the center of a grounded, conducting spherical shell of inner radius R. Recall the potential for a azimuthally uniform potential can be written as $V(r,\theta)=\sum_{\ell}\left(a_\ell r^\ell+\frac{b_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$ where $P_0=1$, $P_1=\cos\theta$ and the potential of an isolated ideal dipole can be written as $V=\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ where p is the dipole moment.

(a) The fact that $V(r=R,\phi)=0$ implies a relationship between a_1 and b_1. Find this relationship.

(b) By matching the potential at $r<<R$ to the dipole form, solve for b_1 in terms of the dipole moment p and use it to obtain a fully explicit expression for $V(r,\theta)$.

(c) Compute the surface charge on the inner surface of the grounded conductor using your answer to part (b). $\boxed{\sigma=-\frac{3p\cos\theta}{4\pi R^3}}$

(d) It turns out that the surface charge you computed in part (c) adds a constant electric field to the field of the electric field of the dipole for $r<R$. Find this additional constant electric field. Hint - consider your expression for the electrical potential. $\boxed{\vec{E}_\sigma=\frac{p\hat{z}}{4\pi\varepsilon_0 R^3}}$

2. In homework you showed $V=\left(as+\frac{b}{s}\right)\cos\phi$ is a possible solution to Laplace's Equation in cylindrical coordinates when the potential has no z-dependence. In this problem you will apply this form to a long, conducting grounded cylinder of radius R in an external electrical field of the form $\vec{E}=E_0\hat{x}$ which implies $V(s>>R)=-E_0 s\cos\phi$.

(a) Find a relation between *a* and *b* from the condition $V(s=R,\phi)=0$.

(b) By matching the potential to the $s>>R$ form find a fully explicit expression for the potential.

(c) Calculate the charge density on the surface of the grounded cylinder as a function of ϕ. $\boxed{\sigma=2\varepsilon_0 E_0\cos\phi}$