Information on the first midterm

The first midterm will be held in class (144 Loomis) on Friday, Feb. 22. The exam is open book, and open notes. There are two problems covering the first two lecture note chapters on Gauss's Law and Potentials. Work the problems in the examination booklets and box your final answer. Partial credit will be given if I can follow your work. No calculator, computer or phone can be used during the exam. If you will arrive on time for the exam you will have 50 minutes to work the exam.

<u>How to prepare for the first hourly.</u>

The best way is to review all of the homework and posted homework solutions with the following in mind.

1. Be familiar with spherical, cylindrical, and Cartesian coordinates and the formulas for surface areas and volumes of spherical and cylindrical systems.

2. Know how to compute the electrical field from Coulomb's Law. Know how to compute the electric field from spherically symmetric and cylindrically symmetric charge distributions and the infinite plane of charge using Gauss's law. Know how to compute the electrical field when these various charge distributions are superimposed.

3. Know how to calculate the curl, and divergence of electric fields in various coordinate systems and know what these quantities mean.

4. Know how to calculate the voltage $V(\vec{r})$ and the charge density $\rho(\vec{r})$ given an electric field $\vec{E}(\vec{r})$. Know how to check your result by taking the gradient of the potential in various coordinate systems.

5. Know how to calculate the energy stored in electrical fields or charge distributions and understand $U_{\text{int}} = \frac{\varepsilon_0}{2}\int 2E_1 \bullet E_2 \, d\tau$

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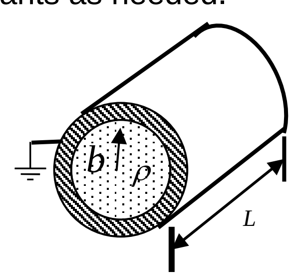
1. A thick, grounded metal shell surrounds an insulated, long cylinder with a uniform charge density ρ. The dimensions (length L and inner shell radius b with $L >> b$) are shown in the <u>very</u> crudely sketched figure. Recall that the electrical field is 0 in a conductor and a grounded conductor has same potential as ∞. Write the answers for all parts of this problem in terms of ρ, b, L cylindrical coordinates and unit vectors, and physical constants as needed.



a. Find $\vec{E}$ everywhere

b. Find the surface charge density on the inside surface of the metal cylindrical shell

c. Find $V(r<b)$. Zero your potential at $r=\infty$. ans $V(s)=-\frac{\rho}{4\varepsilon_0}\left(s^2-b^2\right)$

d. Compute the work required to assemble the charges from infinity using $U=\frac{\varepsilon_0}{2}\int \vec{E}\bullet\vec{E}\, d\tau$. ans $U=\frac{\pi\rho^2 L b^4}{16\varepsilon_0}$

e. Now find the work required to assemble the charges from infinity using $U=\frac{1}{2}\int \rho\, V\, d\tau$.

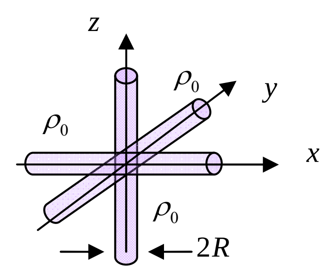
2. Consider an infinitely long cylinder with a radius R carrying a uniform charge density ρ_0 which is aligned along the z-axis. Unless directed otherwise, write the answers for all parts of this problem in terms of ρ_0, R, cylindrical coordinates and unit vectors, and physical constants as needed.

(a) Use Gauss's law to compute the electrical field as a function of s for $s<R$.

(b) Integrate your field to compute the potential $V(s)$ relative to the origin for $s<R$.

(c) Check your result for part (b) by showing $\vec{E}=-\vec{\nabla}V$.

We now add two more infinitely long cylinders of radius R and uniform charge density ρ_0. Hence there is a cylinder along the x-axis, along the y-axis, and along the z-axis as crudely sketched below:



(d) Use superposition to write the potential $V(x,y,z)$ points which are in the **overlap region** of all three cylinders. This is most easily done by writing the potential function for each cylinder in Cartesian coordinates.

ans $V=-\frac{\rho_0}{2\varepsilon_0}\left(x^2+y^2+z^2\right)$

(e) Find the electrical field in the **overlap region** by taking the gradient of your answer to part (d).

(f) Show that the electrical field of part (e) is the same as the field inside of a uniformly charged sphere which carries a charge density ρ_{sphere} which is centered on the origin. Find the ratio of the charge density of the sphere to the cylinder ($\rho_{\text{sphere}}/\rho_0$) which insures the electrical fields are the same. ans $\rho_{\text{sphere}}=3\rho_0$