Physics 435
Spring 2009

Homework #10

1) An infinite wire which passes through the origin and lies parallel to the z axis carries a current I A magnetic material with $\mu = \kappa\mu_0$ is present in the region $s > a$. Answer all parts of this problem in terms of $a, I, \kappa,$ and μ_0.

   a) Find $\vec{H}(s)$ and $\vec{B}(s)$ in all regions.
   b) Find the bound currents $\vec{K}_b$ and $\vec{J}_b$ everywhere
   c) Find the total enclosed current (free and bound) within a circle with $s > a$ and check that your $\vec{B}(s > a)$ is consistent with Ampere's law with this current.

2) Consider an infinitely long solenoid of radius b. A magnetic material exists from $a < s < b$. The magnetization is of the form $\vec{M} = M_0\hat{z}$ in the region $a < s < b$. Air is everywhere else. We define 3 regions: #1 $r < a$, #2 $a < r < b$ and #3 $b < r$. The only free current is a free surface current $\vec{K} = K\hat{\phi}$ at radius b. At *s* = *b* there is also a bound surface current $\vec{K} = K'\ \hat{\phi}$. Another bound surface current exists at *s* = *a*. Answer all parts of this problem in terms of *a*,*b*, *K* , K', and physical constants such as μ_0

   a) Find $\vec{M}$ in terms of *a*,*b*, *K* , K', and physical constants such as μ_0.
   b) Find $\vec{K}(s = a)$
   c) Find $\vec{B}_{\#3}, \vec{B}_{\#2},$ and $\vec{B}_{\#1}$ in terms of *K* and *K'*.
   d) Find $A_{\#3}(s), A_{\#2}(s),$ and $A_{\#1}(s)$ in terms of *K* ,*K'* using the $\oint \vec{A}\bullet d\vec{\ell} = \int \vec{B}\bullet d\vec{a}$ method.
   e) Show that A is continuous at r = a and r = b
   f) Verify the discontinuity BC at the boundary s=a

$$\frac{1}{\mu_o}\left(\frac{\partial\vec{A}_>}{\partial n}\Big|_{\text{interface}} - \frac{\partial\vec{A}_<}{\partial n}\Big|_{\text{interface}}\right) = -\left(\vec{K}_b + \vec{K}_f\right)$$

3) Consider a permanent magnet in the shape of a long cylinder of radius *R* with a position dependent magnetization of $\vec{M} = \beta s\,\hat{z}$ that only exists in the magnet. There are no free currents anywhere.

a) Compute the bound current density $(\vec{J})$ and all bound surface currents $(\vec{K})$.

b) Use Ampere's law to compute $\vec{B}$ everywhere and show your answer implies $\vec{H} = 0$ everywhere . Hint—assume that $\vec{B} = 0$ outside of the magnet and place one side of your Ampere loop outside the magnet.

c) Verify the boundary condition $\vec{B}_> - \vec{B}_< = \mu_o\left(\vec{K}_{free} + \vec{K}_{bound}\right)\times\hat{s}$ at $s = R$.

4) The magnetic field of a spherical permanent magnet of radius R with $\vec{M} = M_0\hat{z}$ and no free currents is given by: $\vec{B}(r<R) = \dfrac{2\mu_0 M_0\hat{z}}{3}$ and

$$\vec{B}(r>R) = \frac{\mu_0 M_0 R^3}{3r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right).$$

a) Show that these magnetic fields satisfy the boundary condition: $\vec{B}_> - \vec{B}_< = \mu_0\vec{K}\times\hat{r}$.

b) Compute $\vec{H}_> \equiv \vec{H}(R+\delta,\theta)$ and $\vec{H}_< \equiv \vec{H}(R-\delta,\theta)$ $(\delta \to 0)$ and comment on which components of $\vec{H}$ are continuous across the boundary at $r = R$. Explain why the tangential $\vec{H}$ component is continuous if there are no free surface currents using the H form of Ampere's law. Which $\vec{B}$ component is continuous: normal or tangential?

c) Find $\int_S \vec{B}\bullet d\vec{a}$ over a surface bounded by a "cap" consisting of the northern hemisphere of spherical shell of radius $a > R$ and a "plate" consisting of the circular disk of radius *a* in the x-y plane. You should get $\int_S \vec{B}\bullet d\vec{a} = 0$ since there are no magnetic charges but I want explicit integrals using the magnetic field forms given in the problem.