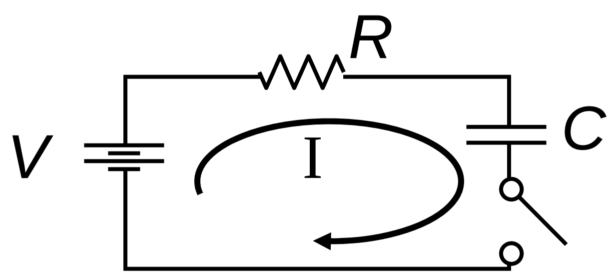
Physics 435
Fall 2010

Homework #4

1. Griffiths problem 2.39

2. A battery supplies a constant potential of V. At time t=0 the switch is closed and the ultimately the capacitor becomes fully charged. Recall $U = \frac{1}{2}CV^2$ is the energy stored in a capacitor which can be thought of as the energy stored in the electromagnetic field contained within the capacitor. Answer all parts of this problem in terms of V, R, and C.



a. Show using Kirchhoff's Laws that the current is $I(t) = I_0 \exp(-t/\tau)$ and find I_0 and τ. One way to do this is to differentiate the KVL equation wrt time and realize that uncharged C acts as short. .

b. By integrating the power supplied by the battery find the total energy supplied by the battery in the act of charging the capacitor in term of C, and V.

c. Find the total energy lost into heat while charging the capacitor by integrating the power dissipated into heat through the resistor

d. Show that the energy supplied by the battery equals the sum of the energy stored in the capacitor and the heat loss through the resistor.

3. Consider an arbitrary, spherically symmetric charge distribution $\rho\left(r\right)$.
   a. Use Gauss's Law to obtain a general form for $V(r)$, the electrical potential due to this charge distribution, in terms of a double integral over $\rho\left(r''\right)$. Assume that $\rho\left(r\right)$ dies off quickly enough that you can zero the potential at infinity.

   b. Check your result by showing $-\varepsilon_0 \nabla^2 V = \rho$. Again remember $\frac{\partial}{\partial x} \int_{\text{const}}^{x} f(x')\, dx' = f(x)$

4. Consider stored energy for a system consisting of two concentric charged, spherical shells. The first shell has a radius R_1 and carries a charge of $+Q$, while the second shell has a radius of R_2 and carries a charge of $+Q$ as well. Assume $R_1 < R_2$.
   a. Compute the stored energy using the "surface version" of $U = \frac{1}{2}\int_V \rho(\vec{r})\mathrm{V}(\vec{r})d\tau$

   b. Compute the stored energy using $U = \frac{\varepsilon_0}{2}\int_0^\infty \vec{E}\cdot\vec{E}\ d\tau$ where the integral is over all space.

.

   c. Show that $U = U_1 + U_2 + U_{\text{int}}$ where U_1 and U_2 are the energy stored in the field of the shells with radius R_1 and R_2 when considered as an isolated systems and $U_{\text{int}} = \frac{\varepsilon_0}{2}\int_0^\infty \left(2\vec{E}_1 \cdot \vec{E}_2\right)\ d\tau$. Be sure to compute U_{int} explicitly.

5. Compute the energy stored in a uniform ball of charge with charge Q and radius R using $U = \frac{\varepsilon_0}{2}\int_{<S} \vec{E}\bullet\vec{E}\ d\tau + \frac{\varepsilon_0}{2}\oint_S V\ \vec{E}\bullet d\vec{a}$ where the surface S is a sphere of radius R. You should get: $U = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R}$. Start by computing $\vec{E}(r < R)$.