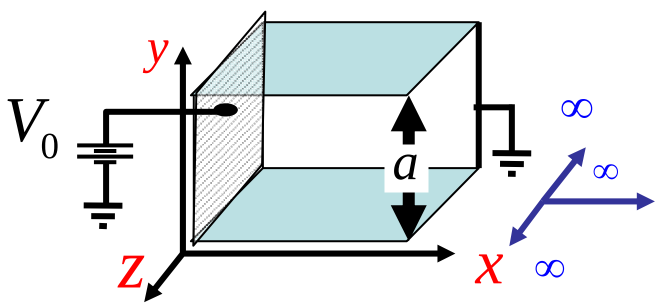
Homework #5

1. A long, conducting cylindrical shell of length L and radius R has a charge of Q. The cylinder is centered with its central axis along the z-axis. Find the total repulsive force that the half cylinder with $x < 0$ exerts on the half cylinder with $x > 0$ using the electrostatic pressure formula.

2. Griffiths problem 3.15

3. This question concerns this problem that we worked out in lecture. Feel free to approximate the potential for the first non-vanishing term in the expansion: $V \rightarrow \frac{4V_0}{\pi}\sin\left(\frac{\pi y}{a}\right)\exp\left(-\frac{\pi x}{a}\right)$



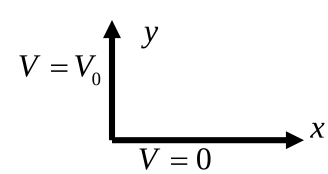
   a. Compute the induced charge density, σ, on the top and bottom plane assuming $V_0 > 0$. See if you can draw some field lines to check your signs.

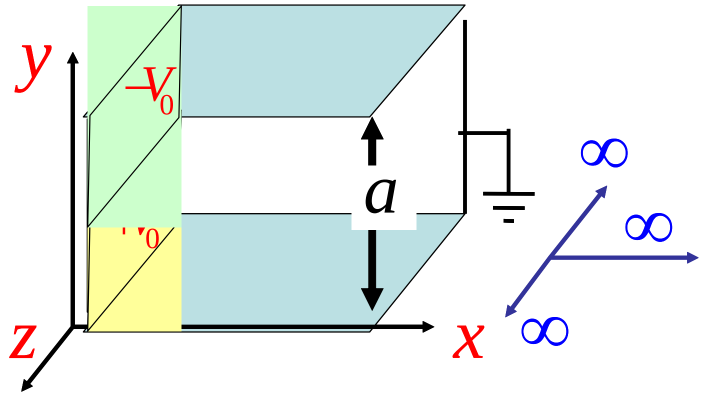
   b. Compute the force per unit z length beyond $x > a$ on the bottom plate for this case using the conductor pressure formula.

4. In lecture we showed that the charge density on the bottom plate for the situation shown in Problem 3 diverged as $\sigma \propto 1/x$ when all terms are considered. We model the corner near $x = y = 0$ as an infinite conducting $y = 0$ plane with potential $V = 0$ and an infinite conducting $x = 0$ plane with potential $V = V_0$.

   a. Show $V(s,\phi,z) = \beta\phi$ satisfies Laplace's Equation in cylindrical coordinates and can match the boundary conditions on the two planes and find β in terms of V_0.

   b. Use your V expression to compute $\sigma(s)$ on the conductor at $\phi = 0$.





5. We next consider modifying the above problem by replacing the potential in the y-z plane with the illustrated, split potential. Assume that $V(0,y) = +V_0$ for $y < a/2$ and $V(0,y) = -V_0$ for $y > a/2$. You need only consider the first non-vanishing expansion of the series: $V(x,y) = \sum_{n=1,2,3\cdots} C_n \sin\left(\frac{n\pi y}{a}\right) \exp\left(-\frac{n\pi x}{a}\right)$ to answer all parts of this problem.

   a. What is n and C_n for the lowest non-vanishing term for this modified potential?

   b. Compute the induced charge density, σ, on the top and bottom plates assuming $V_0 > 0$. See if you can draw some field lines to check your signs.