

Homework #6

- 1) Write the first 2 non-vanishing terms for the $r > R$ potential due to a charge distribution of the form $\sigma = +k$ for $0 < \theta < \pi/2$ and $\sigma = -k$ for $\pi/2 < \theta < \pi$.
- 2) The potential on the surface of a charged, spherical shell of radius R is given by $V(R) = \beta \cos^2 \theta = \frac{\beta}{3} P_0(\cos \theta) + \frac{2\beta}{3} P_2(\cos \theta)$.
 - (a) Calculate the surface charge density on the shell required to create this potential. Hint -- you will need to find the $\hat{r} \cdot \vec{E}$ both inside and outside of the shell using the spherical Laplace equation solutions.
 - (b) Is there a stable equilibrium point for a point-charge placed within the sphere? This is an example of Earnshaw's Theorem. Hint--write the potential for a point within the sphere in Cartesian coordinates.
- 3) Recall the problem of a charge distribution of the form $\sigma = k \cos^2 \theta$ glued to an insulating spherical shell of radius R discussed in lecture. In this problem we will compute the electrostatic pressure on an infinitesimal patch of charge on the "north pole" at $\theta = 0$ in two ways. Feel free to use any and all of the results obtained in lecture to help you. Hint--in computing the force on a patch of charge it is important to remove the field due to the charge itself.
 - a) Compute the pressure using the $V(r > R) = \sum_{\ell=0,1,2} \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos \theta)$ expression.
 - b) Check your result by computing the pressure using the $V(r < R) = \sum_{\ell=0,1,2} A_\ell r^\ell P_\ell(\cos \theta)$ expression.
- 4) Griffiths Problem 3.23. Hints: The azimuthal part of the solutions is sinusoidal with an integral constant of separation to insure that $\Phi(\phi + 2\pi) = \Phi(\phi)$. The comment concerning the infinite line solutions with $S \propto \ln s$ is relevant for the case where $\Phi(\phi)$ is a constant. Try a power law solution for $S(s)$ associated with the other sinusoidal $\Phi(\phi)$ possibilities.

5) Expand $\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{r \sqrt{1 - \frac{2\mathbf{r}' \cdot \mathbf{r}}{r} + \left(\frac{r'}{r}\right)^2}} = \frac{1}{r} \left(1 + \left[\left(\frac{r'}{r}\right)^2 - \frac{2\mathbf{r}' \cdot \mathbf{r}}{r} \right] \right)^{-1/2}$ up to and including the

$\frac{1}{r} \left(\frac{r'}{r}\right)^2$ terms using the binomial expansion:

$(1+x)^N \approx 1 + Nx + \frac{N(N-1)}{2}x^2 + \dots$. Compare to the “generator” function

expansion to show that $P_2(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}') = \frac{3(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}')^2 - 1}{2}$.

6) Consider a charge distribution of the form $\rho = \rho_0 z \exp(-\beta[x^2 + y^2 + z^2])$.

Find the dipole moment $\vec{p} = \int \rho \vec{r}' d\tau'$ and use it to obtain an approximate expression for the potential far from the origin. Hint—these integrals are useful in either Cartesian or spherical coordinates

Gaussian full integrals

$$I_n = \int_{-\infty}^{+\infty} dx x^n \exp(-\beta x^2) ; I_{n+2} = -\frac{\partial I_n}{\partial \beta} ; I_0 = \sqrt{\pi} \beta^{-1/2}$$

$$I_2 = \sqrt{\pi} \left(\frac{1}{2}\right) \beta^{-3/2} ; I_4 = \sqrt{\pi} \left(\frac{1}{2}\right) \left(\frac{3}{2}\right) \beta^{-5/2} ; I_6 = \sqrt{\pi} \left(\frac{1}{2}\right) \left(\frac{3}{2}\right) \left(\frac{5}{2}\right) \beta^{-7/2}$$