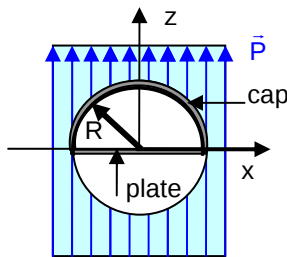


Homework #7

- 1) An infinite line of charge with a free linear charge density of λ passes through the center of a dielectric in the form of a thick cylindrical shell of inner radius a and outer radius b . This dielectric has a position dependent ϵ of the form $\epsilon(a < s < b) = \beta/s$. In all other regions there is vacuum
 - a) Find $\vec{E}$ everywhere using Gauss's law in the form $\oint \vec{D} \cdot d\vec{a} = Q_f$
 - b) Find the surface charge bound charge densities on the inside and outside, ρ_b , and the total bound charge per unit length by computing the polarization density $\vec{P}$ using the electric field you calculated in part (a). Confirm that the total bound charge is zero
 - c) Find $\vec{E}$ everywhere using Gauss's law in the form $\epsilon_0 \oint \vec{E} \cdot d\vec{a} = Q$ where Q includes all (free and bound) charge that you computed in part b).



- 2) Consider a spherical electret with a uniform polarization density of $\vec{P} = P_0 \hat{z}$. A spherical cavity of radius R , centered on the origin is cut out of the electret. There are no free charges anywhere. Feel free to use these results concerning the potential due to a glued charge given in the Laplace chapter.

$$V(r < R) = \frac{\tilde{\sigma}_\ell P_\ell(\theta)}{\epsilon_0 (2\ell + 1)} R \left(\frac{r}{R} \right)^\ell ; V(r > R) = \frac{\tilde{\sigma}_\ell P_\ell(\theta)}{\epsilon_0 (2\ell + 1)} R \left(\frac{R}{r} \right)^{\ell+1}$$

- a) Calculate the bound surface charge density.
- b) Calculate $\vec{E}$ inside the cavity.
- c) Find $\oint_S \vec{D} \cdot d\vec{a}$ over a surface bounded by a “cap” consisting of northern hemisphere of cavity and a “plate” consisting of the circular disk of radius R in the x-y plane. Let the cap have a radius infinitesimally larger than R so it just encloses the bound charge. You should get $\oint_S \vec{D} \cdot d\vec{a} = 0$ since there are no free charges but I want an explicit integral where $\vec{D}$ is constructed from $\vec{P}$ and $\vec{E}$ and you separately compute $\oint_{\text{cap}} \vec{D} \cdot d\vec{a}$ and $\oint_{\text{disk}} \vec{D} \cdot d\vec{a}$ and show that they cancel.

- 3) Consider a long cylinder of length L where L is much larger than any radial dimensions. There is a metal conductor for $s < a$ that carries a free charge of Q . There is a thin conducting shell at $s = b$ that carries a free charge of $-Q$. A dielectric with a dielectric constant ϵ exists from $a < s < b$.
- Use $U' = \int_{\text{all space}} \frac{\vec{D} \cdot \vec{E}}{2} d\tau$ to compute the work required to assemble the free charges on the metal cylinder and shell.
 - Calculate the voltage difference $V(a) - V(b)$.
 - Check your answer to part a) using Griffiths Eq. 2.43
 - Now calculate the work required to assemble the bound as well as free charges using $U = \frac{\epsilon_0}{2} \int \vec{E} \cdot \vec{E} d\tau$.
 - Check your result to part d) using $U = \frac{1}{2} \int \rho V d\tau$ where we include both free and bound charges
- 4) Griffith's problem 4.27 [Just calculate it using (4.55)]
- 5) Consider a sphere of radius R consisting of a class A dielectric with a dielectric constant ϵ . A uniform (free) charge density, ρ , is present in the sphere as well.
- Compute the electrical field for $r < R$ and $r > R$.
 - Separately compute the electrical field contribution due to the free charges and the bound charges for $r < R$ and $r > R$. Is your electrical field continuous across the $r=R$ boundary? Explain any discontinuity.
- 6) Griffith's problem 4.28
- 7) a) Obtain an expression for the bound state volume density, ρ_b , in terms of the free charge volume density, ρ_f , the electric field, and the gradient of the susceptibility χ . Use this relation to answer the following questions.
- Show that the only way to get a bound charge volume density within a Class A dielectric is to have a free charge volume density within the dielectric and non-zero susceptibility.
 - Under what circumstances will there be a bound charge volume density within a dielectric in the absence of any free charge volume density.