Homework #8

1) A current carrying coil is in the form of an *n*-sided polygon which is inscribed within a circle of radius a so that all n corners lie on the circle. Find the ratio of the B-field in the center of the polygon to the B-field in the center of a circular coil of radius a carrying the same current. Show the fields agree in the limit $n \to \infty$.

2) Consider an infinite current sheet on the $z = 0$ plane which carries a uniform surface current : $\vec{K} = K_0 \hat{y}$. Work out the magnetic field in the $z > 0$ and $z < 0$ region using the Biot-Savart Law rather than Ampere's Law.

3) Using the Biot-Savart Law to compute $\vec{B}$ at the origin for an infinite solenoid of radius R centered along the z-axis carrying a surface current $\vec{K} = K\hat{\phi}$. Show that you get $\vec{B} = \mu_0 K \; \hat{z}$ which is the same as the Ampere's Law result obtained in Lecture using the assumption: $\vec{B}(s > R) = \vec{0}$. Hint -- the relevant integral can be found in http://integrals.wolfram.com/ .

4) Find the pressure on a patch of surface current that creates a B-field of $\vec{B} = B_0 \hat{z}$ on the inside of an infinite, air core solenoid. Check your calculation by comparing $\vec{B}_{\text{other}}$ on either side of the surface current patch and specify the direction of the pressure. Remember when computing the force on a small patch it is important to use $\vec{B}_{\text{other}}$ which is the magnetic full magnetic field minus the part of the magnetic field due to the surface current passing through the patch itself. A useful magnetic force expression for surface currents is $d\vec{F} = \vec{K} \times \vec{B}_{\text{other}} da$ and the B-field for a current plane can be written as $\vec{B}_{>,<} = \pm \frac{\mu_0 \vec{K} \times \hat{\eta}}{2}$. Note the analogy between the pressure of this magnetic field and the pressure due to an electrical field on a conductor.

5) Consider an azimuthally directed surface current of the form $\vec{K} \propto \hat{\phi}$ on a sphere of radius R. This is a generalization of Spinning Ball of Charge w/ Magnetic Scalar Potential example worked out in lecture. You design the current such that $\vec{B} = -\vec{\nabla} V_{mag}$ where $V_{mag}(r < R) = Ar^2(3\cos^2\theta - 1)$ and $V_{mag}(r > R) = D\ r^{-3}(3\cos^2\theta - 1)$.

(a) Write $\vec{B}(r < R)$ and $\vec{B}(r > R)$ in terms of $A, D, r,$ and θ.

(b) The $\vec{B}_{>} - \vec{B}_{<} = \mu_0 \vec{K} \times \hat{\eta}$ BC implies that that the radial component of the magnetic field is continuous at $r = R$. Use this to find the constant D in terms of A and R.

(c) Find the required current $\vec{K}$ using the discontinuity in the $\hat{\theta}$ component of $\vec{B}(r = R)$ at $r = R$ implied by the BC in terms of A, R, θ and physical constants. Use a sketch of the currents and fields to argue that $\vec{K}$ is distributed such that the B-field inside the sphere for a point on the x-y plane is radially outwards for A>0 and disappears at the origin.