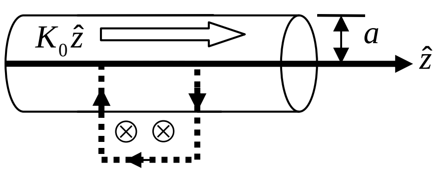
Homework #9

1) Consider a surface current of the form $\vec{K} = k\,\hat{s}$ on the z=0 plane and a thin annulus in the region $a < s < a + \Delta$.

   a) Calculate the total current flowing into and out of the annulus and use these currents to compute $\partial Q / \partial t$ where Q is the total charge in the annulus.
   b) Use the result of part a) to compute $\partial\sigma/\partial t$ for the annulus in terms of k and Δ and work to lowest non-vanishing order in Δ.
   c) Show your result is consistent with the "surface" version of the continuity equation $\partial\sigma / \partial t + \vec{\nabla}\cdot\vec{K} = 0$.

2) Calculate the pressure on a patch of surface current at an angle θ from a spinning ball of radius R. Recall $\vec{B}\left(|\vec{r}| < R\right) = \dfrac{2\mu_0 R\sigma}{3}\omega\hat{z} \equiv B_0\hat{z}$ and write your answer in terms of B_0 which is the magnetic field within the ball. Note the pressure only involves the force component on the patch which lies normal to the ball's surface. Although the electrical pressure is easy to calculate, just calculate the magnetic pressure on the surface.



3) A long, thin cylindrical shell of radius a carries a uniform surface current of $\vec{K} = K_0\hat{z}$.
   a) Calculate the magnetic field for $s < a$ and $s > a$ using Ampere's law.

   b) Verify that your answer to part a) satisfies the boundary condition $\vec{B}_> - \vec{B}_< = \mu_0\vec{K}\times\hat{\eta}$

   c) Use the illustrated path to compute the vector potential using $\oint\vec{A}\cdot d\vec{\ell} = \int B_\phi da$ and check your result using $\vec{\nabla}\times\vec{A} = \vec{B}$. Hint assume $\vec{A} \propto \hat{z}$. Is your $\vec{A}$ expression in the Coulomb gauge?

4) Construct the approximate vector potential for a loop of radius R carrying a current of I centered at the origin for a point near $\vec{r} = (x \quad y \quad z)$ in the limit $x << R, z$ and $y << R, z$. Begin by expanding $|\vec{r} - \vec{r}\,'|^{-1}$ to first order in x and y in the integral form for $\vec{A}$. Use your approximate $\vec{A}$ to compute $\vec{B}(0,0,z)$ which should agree with the Biot-Savart calculation done in lecture and should agree with $\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$ in the limit $z >> x, y, R$.

5) Consider a surface current $\vec{K} = K_0 \hat{x}$ flowing on an infinite *z=0* plane. From $\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{K} da}{\mathscr{r}}$ and the symmetry of this problem we know $\vec{A}(z) = A(z)\,\hat{x}$. Since one can always add a constant to $\vec{A}$, we can arrange it so that $\vec{A}(z=0) = 0$. Use $\oint \vec{A} \cdot d\vec{\ell} = \int \vec{B} \cdot d\vec{a}$ to find $\vec{A}(z<0)$ and $\vec{A}(z>0)$.