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Midterm 2 Information

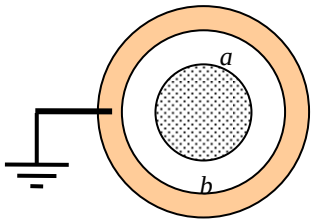
The second midterm (covering HW5+HW6) will be held in class on March 15. The exam is open book, and open notes. There will be two problems. No calculator, computer or phone can be used during the exam.

How to prepare for the second midterm.

The best way is to review all of the homework (HW5 & HW6) and posted homework solutions and the enclosed problems with the following in mind:
1. Be familiar the physics of conductors and know how to evaluate the surface charge and pressure on a conductor. Know how to compute the net force on a piece of a conductor by integrating the surface pressure.
2. Be familiar with the separation of variable solution to Laplace's Equation in spherical, and cylindrical coordinate coordinates. (Cylindrical was covered in a homework problem).
3. Understand how to apply boundary conditions to eliminate unknowns in separation of variable solutions. These include finiteness, constant or zero potentials for conductors, and derivative discontinuity for surface charge glued on insulated surfaces.
4. Know how to apply orthogonal functions to develop infinite series solutions to the Laplace Equation in spherical and cylindrical coordinate coordinates

Here is a somewhat difficult problem to help you review for the second midterm.

Midterm 2 Review Problem



A

A charge density of the form $\sigma = \sigma_0(3\cos^2\theta - 1)$ is glued to the surface of an insulating sphere of radius a. We also have a concentric, grounded, conducting, spherical shell of inner radius b. Assume the potential for this charge distribution can be written as $V(r) = (3\cos^2\theta - 1)(Ar^2 + B/r^3)$. Use A and B for $V(a < r < b)$ and A' and B' for $V(r < a)$.

a) Write *B* in terms of *A* using the fact that the outer sphere is grounded and write the outer potential entirely in terms of *A*.

b) Argue that *B'* must vanish based on an important boundary condition.

c) Express *A'* in terms of *A* using continuity of *V* at *r* = *a*. $\boxed{A' = A(1 - b^5/a^5)}$

d) Solve for *A* in terms of σ_0 using the discontinuity of $\partial V/\partial r$ at *r* = *a*.

$$\boxed{V(a < r < b) = -\frac{a^4\sigma_0}{5b^5\varepsilon_0}(3\cos^2\theta - 1)(r^2 - b^5/r^3)}$$

e) Find the surface charge density on the surface *r* = *b*

$$\boxed{\sigma_b = -\frac{a^4\sigma_0}{b^4}(3\cos^2\theta - 1)}$$

f) Calculate the electrostatic F_z on the "northern hemisphere "of the inner surface of the grounded shell. What is the F_z on the entire (north + south) inner surface? Is there a non-zero F_z on the outer surface of the northern hemisphere?

$$\boxed{F_z^{\text{northern}} = -\frac{a^8\pi\sigma_0^2}{2\varepsilon_0 b^6}}$$