Information on the third midterm

The third midterm will be held in class (144 Loomis) on Friday, April 26 covering HW7-10. The exam is open book, and open notes. There are two problems covering electrostatics in materials, magnetostatics and magnetostatics in materials. Work the problems in the examination booklets and box your final answer. Partial credit will be given if I can follow your work. No calculator, computer or phone can be used during the exam. If you will arrive on time for the exam you will have 50 minutes to work the exam.

<u>How to prepare for the third hourly.</u>

The best way is to review all of the homework and posted homework solutions with the following in mind.

1. Be familiar with how to calculate bound (volume and surface) charges from the polarization and bound (volume and surface) currents from the magnetization.
2. Know how to calculate U and U' from $\vec{E}$ & $\vec{D}$ as well as by using ρ and V.
3. Be familiar with the concept of frozen-in polarization for electrets and frozen-in magnetization in permanent magnets.
4. Be able to calculate magnetic fields from the Biot-Savart and Ampere law.
5. Be very familiar with the magnetic solenoid, and current sheet.
6. Understand the relationship between the current, surface current $\vec{K}$, and volume current $\vec{J}$, how to calculate them from charge densities and velocities and how to convert between them.
7. Understand how surface charges and currents lead to discontinuities in the electric fields and potentials and magnetic fields and potentials.
8. Be able to calculate the magnetic vector potential from the magnetic field using the "flux" method and be able to check your answer using the curl of the magnetic field.
9. Be familiar with the use of the auxiliary fields $\vec{H}$ and $\vec{D}$ and their use in computing polarization, magnetization, bound charges and bound currents.
10. Be able to calculate the magnetic pressure on surfaces carrying $\vec{K}$.

1) A metal sphere with a radius a that carries a free charge of Q. There is a thin conducting shell at $r = b$ where $b > a$ that carries a free charge of -Q so that $\vec{E}(r > b) = 0$. A material with a dielectric constant ε exists from $a < r < b$. Answer all parts of this problem in terms of a, b, Q, ε, physical constants , and spherical coordinates and unit vectors, as needed.

a) Find $\vec{E}(a < r < b)$. Be very careful since essentially all of the rest of this problem depends on getting the correct electric field.

b) Use $U' = \int_{\text{all space}} \frac{\vec{D} \bullet \vec{E}}{2} 4\pi r^2 dr$ to compute the work required to assemble the free charges. $\boxed{U' = \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right)}$

c) Calculate the voltage difference *V(a) – V(b).*

d) Check your answer to part b) using $U' = \frac{1}{2}\int \sigma_f \ V \ da$

e) Use $U = \int_{\text{all space}} \frac{\varepsilon_0 \vec{E} \bullet \vec{E}}{2} 4\pi r^2 dr$ to compute the work required to assemble the total charge on the metal sphere and dielectric shell. $\boxed{U = \frac{\varepsilon_0 Q^2}{8\pi\varepsilon^2}\left(\frac{1}{a} - \frac{1}{b}\right)}$

f) Check your answer to part e) using $U = \frac{1}{2}\int \sigma \ V \ da$ where σ now includes both free and bound charge. Hint $\sigma_f + \sigma_b = \varepsilon_0 \hat{\eta} \cdot \vec{E}$

2) A permanent magnet is in the shape of a thick, long cylindrical shell and carries a magnetization of $\vec{M} = M_0 \hat{z}$ for $a < s < b$ and zero otherwise. There are no free currents anywhere. Answer all parts of this problem in terms of a, b, M_0, physical constants , and cylindrical coordinates and unit vectors, as needed.

a) Find the bound surface current at $s = a$ and $s = b$. Be sure to indicate the direction of the two $\vec{K}$ vectors

b) Find $\vec{B}$ in the regions $s < a$, $a < s < b$, and $s > b$. Hint -- this is easiest using $\vec{H}$ but other simple methods exist.

c) Use $\oint \vec{A} \bullet d\vec{\ell} = \int \vec{B} \bullet d\vec{a}$ to find $\vec{A}(s < a)$ and $\vec{A}(a < s < b)$. Feel free to assume that $\vec{A} \propto \hat{\phi}$. $\boxed{\vec{A}(a < s < b) = \frac{\mu_0 M_0}{2}\left(s - \frac{a^2}{s}\right)\hat{\phi}}$

d) Show your vector potentials satisfy the boundary condition

$$\left[\frac{\partial \vec{A}}{\partial s}\right]_{>} - \left[\frac{\partial \vec{A}}{\partial s}\right]_{<} = -\mu_0 \vec{K}(a).$$