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Extra Final Study Problem

Recall $V = \sum_{m=1}^{\infty} \left(a_m s^m + \frac{b_m}{s^m} \right) \sin m\phi$ solves Laplace's Eq. in cylindrical coordinates. The orthogonality of the sine function is $f(\phi) = \sum_{n=1}^{\infty} a_n \sin n\phi$ with $a_n = \frac{1}{\pi} \int_0^{2\pi} f(\phi)\ \sin n\phi\ d\phi$.

Consider the case where $V(0 < \phi < \pi) = V_0$ and $V(\pi < \phi < 0) = -V_0$ on a long cylinder of radius r.

a) Find the only m value in the series that gives you a non-zero electric field at the origin.

b) Find the $\vec{E}$ field at the origin in terms of V_0 and r in Cartesian coordinates.

$$\boxed{\vec{E} = -\frac{4V_0}{\pi r}\hat{y}}$$

c) Now consider the $\sin 2\phi$ solution for $s < r$. Find $\vec{E}$ for $s < r$ in Cartesian coordinates in terms of a_2 and show $\vec{\nabla} \bullet \vec{E} = 0$.

d) Now consider the $\sin\phi$ solution for a grounded spherical shell of radius R. Find σ on the inside surface of the grounded shell in terms of $a_1 \neq 0$ and R. $\boxed{\sigma = 2a_1\varepsilon_0 \sin\phi}$