Additional Final Study Problems

Consider a spherical permanent magnet of radius R which carries a magnetization of $\vec{M} = M_0\hat{z}$. Write all answers in terms of R, M_0, physical constants,

(a) Find the bound currents $\vec{K}_b$ and $\vec{J}_b$. $\boxed{\vec{K}_b = M_0 \sin\theta\ \hat{\phi}}$

(b) Compute the dipole moment $\vec{m}$ using $\vec{m} = \hat{z}\int \pi\left(R\sin\theta\right)^2 dI$ $\boxed{\vec{m} = \frac{4\pi R^3}{3} M_0\hat{z}}$

(c) The scalar magnetic potential is of the form $V' = \left(ar + \frac{b}{r^2}\right)\cos\theta$. Find *a* and *b* using magnetic discontinuity equation $\vec{B}_> - \vec{B}_< = \mu_0\vec{K}\times\hat{r}$. $\boxed{\mathrm{b} = \frac{\mu_0 M_0 R^3}{3}}$

(d) Is your $\vec{m}$ result of part (b) and your *b* expression from part (c) consistent with $V_>' = \frac{\mu_0 \vec{m}\bullet\hat{r}}{4\pi r^2}$?

(e) Use $\vec{A}_> = \frac{\mu_0 \vec{m}\times\hat{r}}{4\pi r^2}$ and $\vec{B} = \vec{\nabla}\times\vec{A}$ to find $\vec{B}_>$ and compare to the $\vec{B}_>$ expression you obtained during part (c). Recall $\vec{\nabla}\times\vec{A} = \frac{\hat{r}}{r\sin\theta}\partial_\theta\left(r\sin A_\phi\right) - \frac{\hat{\theta}}{r}\partial_r\left(r\ A_\phi\right)$.

(f) Find $\vec{B}\left(r < R\right)$ in Cartesian coordinates. $\boxed{\vec{B}_< = \frac{2\mu_0 M_0}{3}\hat{z}}$

(g) Now imagine drilling a small, cylindrical hole through the spherical, permanent magnet such that $\vec{M}\left(s < a\right) = 0$ where $a \ll R$. Find $\vec{B}$ at the origin in terms of M_0 by superimposing the field that you found in part (e) with the solenoidal field around the cylindrical hole. $\boxed{\vec{B}(0) = -\frac{\mu_0 M_0}{3}\hat{z}}$