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Additional Midterm 3 Problems

1) An unusual electret is in the shape of a long cylindrical shell with inner radius a and outer radius b. It has a polarization density given by $\vec{P} = P_0\hat{s}$ where P_0 is a constant. No free charges are present.
   a) Calculate the bound charge density ρ_b within the electret and the bound charge surface density on the inner and outer radii, and check your result by showing the total bound charge is zero.

   $$\rho_b = -\frac{P_0}{s}; \quad \sigma_a = -P_0; \quad \sigma_b = P_0$$

   b) Compute the $\vec{E}$ field everywhere: $s < a,\ a < s < b,$ and $s > b$.

   c) Compute the $\vec{D}$ field everywhere: $s < a,\ a < s < b,$ and $s > b$.

   d) Give the cylinder a length ℓ and compute $U = \frac{1}{2}\int V\rho d\tau$. Check using $U = \frac{\varepsilon_0}{2}\int \vec{E} \bullet \vec{E} d\tau$

   $$U = \frac{\pi \ell P_0^2}{2\varepsilon_0}\left(b^2 - a^2\right)$$

2) A uniform surface charge density of σ is glued on to a long cylinder of radius R with a central axis along the z-axis. The cylinder rotates with a constant angular velocity of $\omega\hat{z}$. Recall $\vec{v} = \vec{\omega} \times \vec{r}$.

   a) Find the magnetic fields in the regions $s < R$ and $s > R$ using Ampere's Law.

   b) Find the vector potential $\vec{A}(\vec{r})$ in the regions $s < R$ and $s > R$. Please use $\oint \vec{A} \cdot d\vec{\ell} = \int_S \vec{B} \cdot d\vec{a}$ to find $\vec{A}(s)$. Check your result using $\vec{B} = \vec{\nabla} \times \vec{A}$

   $$\vec{A} = \hat{\phi}\frac{\mu_0 \sigma R \omega\, s}{2} \text{ for } s < R \ \&\ \vec{A} = \hat{\phi}\frac{\mu_0 \sigma \omega R^3}{2s} \text{ for } s > R$$