

Discussion 14

1) Consider a time varying magnetic field of the form $\vec{B} = B(s, t) \hat{z}$.

a) Apply the integral form of Faraday's Law $\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$ to a circle

of radius s to show: $\vec{E}(s) = -\frac{\hat{\phi}}{s} \int_0^s s' \frac{\partial B(s')}{\partial t} ds'$. Use the fundamental

theorem of calculus to show this $\vec{E}(s)$ satisfies: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

b) Construct $\int \vec{B} \cdot d\vec{a}$ for the northern hemisphere of a sphere of radius s where $d\vec{a} = \hat{r} 2\pi s^2 \sin\theta d\theta$ to show that you get the same $\int \vec{B} \cdot d\vec{a}$ as that you computed in part a). Hint – use the substitution $s' = s \sin\theta$. What would be the correct $d\vec{a}$ to get the same $\int \vec{B} \cdot d\vec{a}$ using the southern hemisphere?

2) Griffiths problem 7.18. For simplicity put the loop edge at $s=a$.