The electrical field within a uniform ball of charge of radius R and charge Q. is $\vec{E}(r < R) = \dfrac{Q\,r}{4\pi\varepsilon_0 R^3}\hat{r}$, its charge density is $\rho = \dfrac{3Q}{4\pi R^3}$, and the stored energy of a ball of charge is $U_{ball} = \dfrac{3}{5}\dfrac{Q^2}{4\pi\varepsilon_0 R}$. We put a cancelling charge of –Q on the surface of the ball so that there is no electrical field for $r > R$. Recall the stored energy in a thin shell of charge is given by: $U_{shell} = \dfrac{Q^2}{8\pi\varepsilon_0 R}$

(a) Calculate the potential function $V(r) - V_\infty$ for this system and use it to compute the stored energy using $U = \dfrac{1}{2}\displaystyle\int_{volume} \rho\, V\, d\tau$

(b) Check your answer by computing the stored energy using the "interference" approach: $U_{ball+shell} = U_{ball} + U_{shell} + \dfrac{\varepsilon_0}{2}\displaystyle\int 2\vec{E}_{ball}\cdot\vec{E}_{shell}\, d\tau$