Homework #2

1) A charge q is placed a distance a above the center of a square with sides of $2a$.

   a) Compute the electric flux Φ by "brute force" integration $\Phi = \int_{\text{square}} \vec{E} \bullet d\vec{a}$. Hint -- the relevant integrals can be found in http://integrals.wolfram.com/ .

   b) Check your answer by using a simple Gauss's Law argument based on putting a charge in the center of a cube.

2) Consider the electric field given by

$$\vec{E} = \frac{\beta x}{\varepsilon_0}\hat{x} \text{ for } |\text{x}| < \frac{W}{2};\ \vec{E} = \frac{W\beta}{2\varepsilon_0}\hat{x} \text{ for } x > \frac{W}{2} \text{ and } \vec{E} = -\frac{W\beta}{2\varepsilon_0}\hat{x} \text{ for } x < -\frac{W}{2}$$

   a) Find $\rho(\vec{r})$ everywhere using Gauss's Law in differential form $\varepsilon_0 \vec{\nabla} \bullet \vec{E} = \rho$.

   b) Obtain this form for $\vec{E}$ using superposition and the Gauss's Law expression $\vec{E} = \pm \frac{\sigma}{2\varepsilon_0}\hat{x}$ for an infinitely wide slab of charge in the y-z plane. Hint-- what is the effective σ for a charged slab with a volume charge density ρ and a thickness T.

3) In Cartesian coordinates it is easy to see that $\vec{\nabla} x = \hat{x}$ and $\vec{\nabla} \bullet (x\hat{x}) = 1$

   a) Show that the $\hat{x}$ unit vector can be written in terms of the cylindrical coordinate unit vector as $\hat{x} = \cos\phi\ \hat{s}$ - $\sin\phi\ \hat{\phi}$ using trigonometry.

   b) Use the gradient expression in cylindrical coordinates and the result of part (a) to verify that $\vec{\nabla} x = \vec{\nabla}(s\cos\phi) = \hat{x}$

   c) Use the divergence expression in cylindrical coordinates to verify that $\vec{\nabla} \bullet (x\hat{x}) = \vec{\nabla} \bullet (s\cos\phi\hat{x}) = \vec{\nabla} \bullet \left(s\cos\phi\left[\cos\phi\,\hat{s} - \sin\phi\,\hat{\phi}\right]\right) = 1$

4. Consider an arbitrary, spherically symmetric charge distribution $\rho(r)$.

   (a) Obtain a general form for $\vec{E}$, the electric field due to this charge distribution in terms of an integral over $\rho(r')$ using Gauss's Law.

   (b) Check your answer to part (a) by taking the divergence of the expression you obtained in part (a) and showing that $\varepsilon_0 \vec{\nabla} \bullet \vec{E} = \rho(r)$. This statement of the Fundamental Theorem of Calculus might be helpful:

$$\frac{\partial}{\partial x} \int_{\text{const}}^{x} f(x')\,dx' = f(x)$$