Physics 435
Fall 2010

Homework #3

1. Griffiths problem 2.21

2. Griffiths problem 2.35

3. Calculate the energy stored in charged sphere of radius R with a charge density which grows as $\rho(r) \propto r^n$ up to R and is zero for *r* > *R*. Let Q be the total charge contained in the charged sphere. Show that U approaches U $\to Q^2/(8\pi\varepsilon_0 R)$ in the limit $n \to \infty$ and explain why it approaches the stored energy for a spherical shell. A quick sketch of $\rho(r) \propto r^n$ for various *n* should make it clear why this happens.

4. Consider an electric field of the form $\vec{E} = \beta\begin{pmatrix} z & z & x+y \end{pmatrix}$
   a. Show that $\vec{\nabla} \times \vec{E} = 0$ so a potential can be found
   b. Compute $V(x,y,z) - V(0,0,0) = -\int \vec{E} \bullet d\vec{\ell}$ by integrating along a straight-line path from $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} x & y & z \end{pmatrix}$
   c. Use an alternative path to compute $V(x,y,z) - V(0,0,0)$.

5. Consider the electrical field $\vec{E} = -\beta y\,\hat{x} + \beta x\,\hat{y}$ which approximates the electrical field on accelerating particles by a betatron. (The betatron – invented at the U of I -- is displayed near 151 Loomis.)

   a. By computing the curl, show that this field cannot be created by a static charge distribution.
   b. Compute $\oint \vec{E} \bullet d\vec{\ell}$ around a counter-clockwise rectangular path with opposite corners at $\begin{pmatrix} 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} a & b \end{pmatrix}$ and show that your answer is consistent with $\oint \vec{E} \bullet d\vec{\ell} = \iint \vec{\nabla} \times \vec{E} \bullet \hat{z}\; dxdy$
   c. Now find the counter-clockwise $\oint \vec{E} \bullet d\vec{\ell}$ around a circle of radius R centered at the origin. This is most easily done by parameterizing the path as $\vec{r}_{path} = \begin{pmatrix} R\cos\phi & R\sin\phi \end{pmatrix}$.
   d. Show that you get the same $\oint \vec{E} \bullet d\vec{\ell}$ even if the circle center is not centered on the origin and argue from the Stokes' theorem why this must be true for this field.