Physics 435
Fall 2013

Homework #6

1) Write the first 2 non-vanishing terms for the $r > R$ potential due to a charge distribution of the form $\sigma = +k$ for $0 < \theta < \pi/2$ and $\sigma = -k$ for $\pi/2 < \theta < \pi$.

2) The potential on the surface of a charged, spherical shell of radius R is given by $V(R) = \beta\cos^2\theta = \frac{\beta}{3}P_0(\cos\theta) + \frac{2\beta}{3}P_2(\cos\theta)$.

   (a) Calculate the surface charge density on the shell required to create this potential. Hint – take the glued charge potential, evaluate it at $r = R$, and equate it to $V(R)$.

   (b) Is there a stable equilibrium point for a point-charge placed within the sphere? This is an example of Earnshaw's Theorem. Hint--write the potential for a point within the sphere in Cartesian coordinates.

3) Recall the problem of a charge distribution of the form $\sigma = k\cos^2\theta$ glued to an insulating spherical shell of radius R discussed in lecture. In this problem we will compute the electrostatic pressure on an infinitesimal patch of charge on the "north pole" at $\theta = 0$ in two ways. Feel free to use any and all of the results obtained in lecture to help you. Hint-in computing the force on a patch of charge it is important to remove the field due to the charge itself.

   a) Compute the pressure using the $V(r > R) = \sum_{\ell=0,1,2} \frac{B_\ell}{r^{\ell+1}} \mathrm{P}_\ell(\cos\theta)$ expression.

   b) Check your result by computing the pressure using the $V(r < R) = \sum_{\ell=0,1,2} A_\ell r^\ell \mathrm{P}_\ell(\cos\theta)$ expression.

4) Griffiths Problem 3.23. Hints: The azimuthal part of the solutions is sinusoidal with an integral constant of separation to insure that $\Phi(\phi + 2\pi) = \Phi(\phi)$. The comment concerning the infinite line solutions with $S \propto \ln s$ is relevant for the case where $\Phi(\phi)$ is a constant. Try a power law solution for *S(s)* associated with the other sinusoidal $\Phi(\phi)$ possibilities.

5) Expand $\frac{1}{\mathcal{r}} = \frac{1}{\left| r\,\hat{r} - r'\hat{r}' \right|} = \frac{1}{r}\left(1 + \left[\left(\frac{r'}{r}\right)^2 - \frac{2r'}{r}\hat{r}'\cdot\hat{r}\right]\right)^{-1/2}$ up to and including the $\frac{1}{r}\left(\frac{r'}{r}\right)^2$ terms using the binomial expansion: $(1+x)^N \approx 1 + Nx + \frac{N(N-1)}{2}x^2 + \cdots$. Compare to the "generator" function expansion to show that $P_2(\hat{r}\bullet\hat{r}') = \frac{3(\hat{r}\bullet\hat{r}')^2 - 1}{2}$.

6) Consider a charge distribution of the form $\rho = \rho_0\, z \exp\left(-\beta\left[x^2 + y^2 + z^2\right]\right)$. Find the dipole moment $\vec{p} = \int \rho\, \vec{r}'\, d\tau'$ and use it to obtain an approximate expression for the potential far from the origin. Hint—these integrals are useful in either Cartesian or spherical coordinates

Gaussian full integrals

$$I_n = \int_{-\infty}^{+\infty} dx\ x^n \exp(-\beta x^2)\ ;\ I_{n+2} = -\frac{\partial I_n}{\partial \beta}\ ;\ I_0 = \sqrt{\pi}\,\beta^{-1/2}$$

$$I_2 = \sqrt{\pi}\left(\frac{1}{2}\right)\beta^{-3/2}\ ;\ I_4 = \sqrt{\pi}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\beta^{-5/2}\ ;\ I_6 = \sqrt{\pi}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\beta^{-7/2}$$