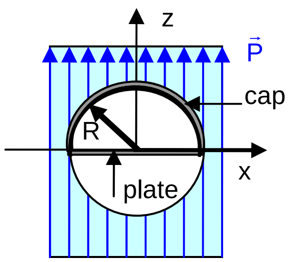
Physics 435
Spring 2014

Homework #7

1) An infinite line of charge with a free linear charge density of λ passes through the center of a dielectric in the form of a thick cylindrical shell of inner radius a and outer radius b. This dielectric has a position dependent ε of the form $\varepsilon(a < s < b) = \beta / s$. In all other regions there is vacuum
   a) Find $\vec{E}$ everywhere using Gauss's law in the form $\int \vec{D} \cdot d\vec{a} = Q_f$
   b) Find the surface charge bound charge densities on the inside and outside, ρ_b , and the total bound charge per unit length by computing the polarization density $\vec{P}$ using the electric field you calculated in part (a). Confirm that the total bound charge is zero
   c) Find $\vec{E}$ everywhere using Gauss's law in the form $\varepsilon_0 \int \vec{E} \cdot d\vec{a} = Q$ where Q includes all (free and bound) charge that you computed in part b).



2) Consider a spherical electret with a uniform polarization density of $\vec{P} = P_0 \hat{z}$. A spherical cavity of radius R, centered on the origin is cut out of the electret. There are no free charges anywhere. Feel free to use these results concerning the potential due to a glued charge given in the Laplace chapter.

$$V(r < R) = \frac{\tilde{\sigma}_\ell P_\ell(\theta)}{\varepsilon_0 \left(2\ell + 1\right)} R \left(\frac{r}{R}\right)^{\ell} \; ; V(r > R) = \frac{\tilde{\sigma}_\ell P_\ell(\theta)}{\varepsilon_0 \left(2\ell + 1\right)} R \left(\frac{R}{r}\right)^{\ell+1}$$

   a) Calculate the bound surface charge density.
   b) Calculate $\vec{E}$ inside the cavity.
   c) Find $\int_S \vec{D} \cdot d\vec{a}$ over a surface bounded by a "cap" consisting of northern hemisphere of cavity and a "plate" consisting of the circular disk of radius R in the x-y plane. Let the cap have a radius infinitesimally larger than R so it just encloses the bound charge. You should get $\int_S \vec{D} \cdot d\vec{a} = 0$ since there are no free charges but I want an explicit integral where $\vec{D}$ is

constructed from $\vec{P}$ and $\vec{E}$ and you separately compute $\int_{\text{cap}} \vec{D}\cdot d\vec{a}$ and $\int_{\text{disk}} \vec{D}\cdot d\vec{a}$ and show that they cancel.

3) Consider a long cylinder of length L where L is much larger than any radial dimensions. There is a metal conductor for $s < a$ that carries a free charge of Q. There is a thin conducting shell at $s = b$ that carries a free charge of -Q. A dielectric with a dielectric constant ε exists from $a < s < b$.
   a) Use $U' = \int_{\text{all space}} \frac{\vec{D}\cdot\vec{E}}{2}\, d\tau$ to compute the work required to assemble the free charges on the metal cylinder and shell.
   b) Calculate the voltage difference *V(a) – V(b).*
   c) Check your answer to part a) using Griffiths Eq. 2.43
   d) Now calculate the work required to assemble the bound as well as free charges using $U = \frac{\varepsilon_0}{2}\int \vec{E}\cdot\vec{E}\, d\tau$.
   e) Check your result to part d) using $U = \frac{1}{2}\int \rho V d\tau$ where we include both free and bound charges

4) Griffith's problem 4.27 [Just calculate it using (4.58)]

5) Consider a sphere of radius R consisting of a class A dielectric with a dielectric constant ε. A uniform (free) charge density, ρ, is present in the sphere as well.
   a) Compute the electrical field for r < R and r > R.
   b) Separately compute the electrical field contribution due to the free charges and the bound charges for r < R and r > R. Is your electrical field continuous across the r=R boundary? Explain any discontinuity.

6) Griffith's problem 4.28

7) a) Obtain an expression for the bound state volume density, ρ_b, in terms of the free charge volume density, ρ_f, the electric field, and the gradient of the susceptibility χ. Use this relation to answer the following questions.

   b) Show that the only way to get to get a bound charge volume density within a Class A dielectric is to have a free charge volume density within the dielectric and non-zero susceptibility.

   c) Under what circumstances will there be a bound charge volume density within a dielectric in the absence of any free charge volume density.