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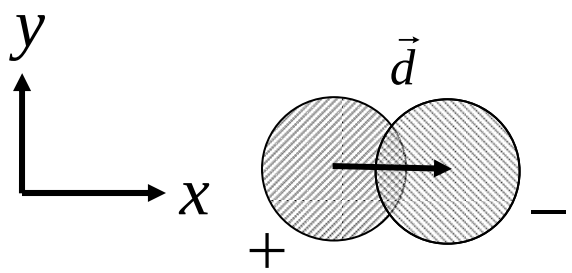
Extra Midterm 1 Review Problems



1. Consider two infinitely long, charged cylinders with central axes parallel to the z direction. Both cylinders have a radius *R.* Their centers are offset by a vector $\vec{d}$ in the x-y plane of the paper as shown. The left cylinder has a uniform charge density of $+\rho_0$ and the right cylinder has a uniform charge density of $-\rho_0$.

   a. Find the magnitude of the electrical field on the central axis of the negative cylinder assuming $|\vec{d}| > R$ as shown.

   b. Find the electrical field in the overlap region in terms of $\rho, R,$ and $\vec{d}$. Hint -- This problem is similar to the overlapping sphere home work. Ans. $\vec{E} = \dfrac{\rho_0\,\vec{d}}{2\varepsilon_0}$

2. Consider an arbitrary, cylindrically symmetric charge distribution of the form $\rho(s)$. Assume the cylinder is very long so there is no z or ϕ dependence.

   a. Use Gauss's law to find the electrical field as function of s in terms of an integral over $\rho(s)$.

   b. Check your expression for E using $\varepsilon_0 \vec{\nabla} \cdot \vec{E} = \rho$. This statement of the Fundamental Theorem of Calculus might be helpful: $\dfrac{\partial}{\partial x} \int_{\text{const}}^{x} f(x')\,dx' = f(x)$

   c. Use your expression for E to obtain a double integral expression for $V(s)$ in the region *s* < *b* where *s* = *b* is the zero for the potential. Ans. $V(s) = -\dfrac{1}{\varepsilon_0}\int_b^s \dfrac{ds'}{s'}\int_0^{s'} s''\rho(s'')\,ds''$

   d. Check your answer to part c. by showing that $V(s)$ satisfies $\nabla^2 V = -\rho/\varepsilon_0$.