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Spring 2014

Midterm 2 Information

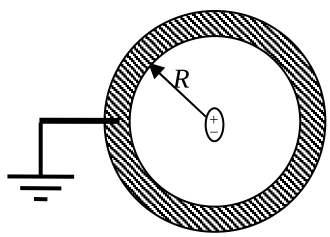
The second midterm (covering HW5+HW6) will be held during lecture on Mar. 21. The exam is open book, and open notes. There will be two problems. No calculator will be necessary. **The final deadline for credit for HW6 is Tuesday, Mar. 18 (< 1pm) so I can post solutions.**

How to prepare for the second midterm.

The best way is to review all of the homework (HW5 & HW6) and posted homework solutions and the enclosed problems with the following in mind:

1. Be familiar the physics of conductors and know how to evaluate the surface charge and pressure on a conductor. Know how to compute the net force on a piece of a conductor by integrating the surface pressure.
2. Be familiar with the separation of variable solution to Laplace's Equation in spherical, and cylindrical coordinate coordinates. (Cylindrical was covered in a homework problem).
3. Understand how to apply boundary conditions to eliminate unknowns in separation of variable solutions. These include finiteness, constant or zero potentials for conductors, and derivative discontinuity for surface charge glued on insulated surfaces.
4. Know how to apply orthogonal functions to develop infinite series solutions to the Laplace Equation in spherical and cylindrical coordinate coordinates

Included are problems that **will be discussed during the Wed, Oct. 19 lecture** to help you prepare for the 2nd midterm.



1. An ideal dipole, with a dipole moment of p, lies in the center of a grounded, conducting spherical shell of inner radius R. Recall the potential for a azimuthally uniform potential can be written as $V(r,\theta)=\sum_\ell\left(a_\ell r^\ell+\frac{b_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$ where $P_0=1$, $P_1=\cos\theta$ and the potential of our isolated ideal dipole can be written as $V=\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ where p is the dipole moment. Write all answers to this problem in terms of p,q,R, spherical coordinates and unit vectors and physical constants as needed.

(a) The fact that $V(r=R,\phi)=0$ implies a relationship between a_1 and b_1. Find this relationship.

(b) By matching the potential at $r < R$ to the dipole form, solve for b_1 in terms of the dipole moment p and use it to obtain a fully explicit expression for $V(r,\theta)$. ans $V = \frac{p}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)\cos\theta$

(c) Compute the surface charge on the inner surface of the grounded conductor using your answer to part (b). ans: $\sigma = -\frac{3p\cos\theta}{4\pi R^3}$

(d) It turns out that the surface charge you computed in part (c) adds a constant electric field in addition to the field of the electric field of the dipole for $r < R$. Find this additional constant electric field. Hint - consider your expression for the electrical potential.

We now add a point charge q to the system so that we have the point charge as well as the dipole in the center of the sphere.

(e) Obtain a fully explicit expression for $V(r,\theta)$ describing the charge plus dipole system.

(f) Compute the surface charge on the inner surface of the grounded conductor using your answer to part (e). ans $\sigma = -\frac{qR + 3p\cos\theta}{4\pi R^3}$

2) In homework you showed $V = \left(as + \frac{b}{s}\right)\cos\phi$ is a possible solution to Laplace's Equation in cylindrical coordinates when the potential has no z-dependence. Assume this form works for the case of a glued charge on the surface of a long, insulated cylinder of radius R that we will discuss.

a) Write separate expression for the potential in the region $s < R$ and $s > R$ that insures that the potential is finite. Hint—either *a=0* or *b=0* for $s < R$ or *V* will be infinite at the origin.

   For $s < R$: $V = as\cos\phi$. For $s > R$: $V = b\cos\phi / s$

b) Use the continuity of *V* at *s* = *R* to eliminate one of the remaining unknown coefficients.

   $b = aR^2$

c) Assume the glued, surface charge density at s = *R* is given by $\sigma = k\cos\phi$. Use the discontinuity in $\partial V / \partial s$ on the surface of the cylinder due to the surface charge density to evaluate the remaining coefficient in terms of $k, R,$ and physical constants as needed. Summarize your answer by writing fully explicit expressions for V in terms of k, s, ϕ, R and physical constants as needed for the two cases $s < R$ and $s > R$.

   $V(s > R) = \frac{kR^2}{2\varepsilon_0 s}\cos\phi \;\&\; V(s < R) = \frac{ks}{2\varepsilon_0}\cos\phi$