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## Del in cylindrical and spherical coordinates

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This is a list of some vector calculus formulae of general use in working with standard coordinate systems.

## Table with the del operator in cylindrical and spherical coordinates

Operation	Cartesian coordinates (x,y,z)	Cylindrical coordinates (ρ,φ,z)	Spherical coordinates (r,θ,φ)
Definition of coordinates		$\begin{bmatrix} x = \rho \cos \phi \\ y = \rho \sin \phi \\ z = z \end{bmatrix}.$ $\boxed{\left[\rho = \sqrt{x^2 + y^2}\right]}$	$\begin{bmatrix} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{bmatrix}.$ $\begin{bmatrix} r = \sqrt{x^2 + y^2 + z^2} \end{bmatrix}$
			$egin{array}{rl}  heta &=& rccos(z/r) = rctan(\sqrt{x^2+y^2}/z) \ \phi &=& rctan(y/x) \end{array}  ight.$
A vector field A	$A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} + A_z \hat{\mathbf{z}}$	$A_{ ho}\hat{ ho} + A_{\phi}\hat{\phi} + A_z\hat{z}$	$A_r \hat{oldsymbol{r}} + A_ heta \hat{oldsymbol{ heta}} + A_\phi \hat{oldsymbol{\phi}}$
Gradient $ abla f$	$\frac{\partial f}{\partial x}\mathbf{\hat{x}} + \frac{\partial f}{\partial y}\mathbf{\hat{y}} + \frac{\partial f}{\partial z}\mathbf{\hat{z}}$	$rac{\partial f}{\partial  ho} \hat{oldsymbol{ ho}} + rac{1}{ ho} rac{\partial f}{\partial \phi} \hat{oldsymbol{ ho}} + rac{\partial f}{\partial z} \hat{oldsymbol{z}}$	$rac{\partial f}{\partial r} \hat{m{r}} + rac{1}{r} rac{\partial f}{\partial  heta} \hat{m{ heta}} + rac{1}{r\sin heta} rac{\partial f}{\partial \phi} \hat{m{\phi}}$
$\begin{array}{c} \textbf{Divergence} \\ \nabla \cdot \mathbf{A} \end{array}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho} \frac{\partial \left(\rho A_{\rho}\right)}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_{\phi}}{\partial \phi} + \frac{\partial A_{z}}{\partial z}$	$\frac{1}{r^2}\frac{\partial\left(r^2A_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$
Curl $ abla  imes \mathbf{A}$	$ \begin{pmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \end{pmatrix} \hat{\mathbf{x}} + \\ \begin{pmatrix} \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \end{pmatrix} \hat{\mathbf{y}} + \\ \begin{pmatrix} \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{pmatrix} \hat{\mathbf{z}} $	$ \begin{pmatrix} \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \end{pmatrix} \hat{\boldsymbol{\rho}} & + \\ \begin{pmatrix} \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \end{pmatrix} \hat{\boldsymbol{\phi}} & + \\ \frac{1}{\rho} \begin{pmatrix} \frac{\partial (\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \end{pmatrix} \hat{\boldsymbol{z}} \end{cases} $	$ \frac{1}{r\sin\theta} \left( \frac{\partial}{\partial\theta} (A_{\phi}\sin\theta) - \frac{\partial A_{\theta}}{\partial\phi} \right) \hat{\boldsymbol{r}} + \frac{1}{r} \left( \frac{1}{\sin\theta} \frac{\partial A_{r}}{\partial\phi} - \frac{\partial}{\partial r} (rA_{\phi}) \right) \hat{\boldsymbol{\theta}} + \frac{1}{r} \left( \frac{\partial}{\partial r} (rA_{\theta}) - \frac{\partial A_{r}}{\partial\theta} \right) \hat{\boldsymbol{\phi}} $
Laplace operator $\Delta f = \nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$ or $\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
$\Delta \mathbf{A} = \nabla^2 \mathbf{A}$	$\Delta A_x \hat{\mathbf{x}} + \Delta A_y \hat{\mathbf{y}} + \Delta A_z \hat{\mathbf{z}}$	$ \begin{pmatrix} \Delta A_{\rho} - \frac{A_{\rho}}{\rho^2} - \frac{2}{\rho^2} \frac{\partial A_{\phi}}{\partial \phi} \end{pmatrix} \hat{\boldsymbol{\rho}} &+ \\ \begin{pmatrix} \Delta A_{\phi} - \frac{A_{\phi}}{\rho^2} + \frac{2}{\rho^2} \frac{\partial A_{\rho}}{\partial \phi} \end{pmatrix} \hat{\boldsymbol{\phi}} &+ \\ & (\Delta A_z)  \hat{\boldsymbol{z}} \end{pmatrix} $	$ \begin{pmatrix} \Delta A_r - \frac{2A_r}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial (A_\theta \sin \theta)}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi} \end{pmatrix} \hat{\boldsymbol{r}} + \\ \begin{pmatrix} \Delta A_\theta - \frac{A_\theta}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial A_r}{\partial \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\phi}{\partial \phi} \end{pmatrix} \hat{\boldsymbol{\theta}} + \\ \begin{pmatrix} \Delta A_\phi - \frac{A_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin^2 \theta} \frac{\partial A_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\theta}{\partial \phi} \end{pmatrix} \hat{\boldsymbol{\phi}} \end{cases} $
Differential displacement	$d\mathbf{l} = dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$	$d\mathbf{l} = d\rho\hat{\boldsymbol{\rho}} + \rho d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{z}}$	$d\mathbf{l} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$
Differential normal area	$d\mathbf{S} = \begin{array}{c} dy dz \hat{\mathbf{x}} + \\ dx dz \hat{\mathbf{y}} + \\ dx dy \hat{\mathbf{z}} \end{array}$	$egin{array}{lll} d{f S} = &  ho d\phi dz {f \hat ho} + \ & d ho dz {f \hat ho} + \ &  ho d ho dz {f \hat ho} + \ &  ho d ho d\phi {f \hat z} \end{array}$	$d\mathbf{S} = r^2 \sin  heta d  heta d \phi \hat{\mathbf{r}} + r \sin  heta d r d \phi \hat{m{ heta}} + r d r d  heta \hat{m{ heta}}$
Differential volume	dv = dxdydz	$dv = \rho d\rho d\phi dz$	$dv = r^2 \sin \theta dr d\theta d\phi$

Non-trivial calculation rules:

1. div grad  $f = \nabla \cdot (\nabla f) = \nabla^2 f = \Delta f$  (Laplacian)

2. curl grad 
$$f = \nabla \times (\nabla f) = 0$$

- 3. div curl  $\mathbf{A} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$
- 4. curl curl  $\mathbf{A} = \nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) \nabla^2 \mathbf{A}$  (using Lagrange's formula for the cross product) 5.  $\Delta fg = f \Delta g + 2\nabla f \cdot \nabla g + g \Delta f$

## Remarks

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- This page uses standard physics notation; some (American mathematics) sources define φ as the angle from the z-axis instead of θ.
- The function atan2(y, x) is used instead of the mathematical function arctan(y/x) due to its domain and image. The classical arctan(y/x) has an image of  $(-\pi/2, +\pi/2)$ , whereas atan2(y, x) is defined to have an image of  $(-\pi, \pi]$ .

## See also

- Orthogonal coordinates
- Curvilinear coordinates
- Vector fields in cylindrical and spherical coordinates

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Categories: Vector calculus | Coordinate systems

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