## Del in cylindrical and spherical coordinates

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(Redirected from Nabla in cylindrical and spherical coordinates)
This is a list of some vector calculus formulae of general use in working with standard coordinate systems.
Table with the del operator in cylindrical and spherical coordinates

| Operation | Cartesian coordinates ( $\mathbf{x}, \mathbf{y}, \mathbf{z}$ ) | Cylindrical coordinates ( $\rho, \varphi, \mathrm{z}$ ) | Spherical coordinates (r, $\boldsymbol{\theta}, \mathrm{\varphi}$ ) |
| :---: | :---: | :---: | :---: |
|  |  | $\left[\begin{array}{llc}x & = & \rho \cos \phi \\ y= & \rho \sin \phi \\ z= & z\end{array}\right]$. | $\left[\begin{array}{llc}x & = & r \sin \theta \cos \phi \\ y= & r \sin \theta \sin \phi \\ z= & r \cos \theta\end{array}\right]$. |
| of coordinates |  | $\left[\begin{array}{ccc}\rho & = & \sqrt{x^{2}+y^{2}} \\ \phi= & \arctan (y / x) \\ z= & z\end{array}\right]$. | $\left[\begin{array}{llc}r= & \sqrt{x^{2}+y^{2}+z^{2}} \\ \theta= & = & \arccos (z / r)=\arctan \left(\sqrt{x^{2}+y^{2}} / z\right) \\ \phi= & \arctan (y / x)\end{array}\right]$ |
| A vector field | $A_{x} \hat{\mathbf{x}}+A_{y} \hat{\mathbf{y}}+A_{z} \hat{\mathbf{z}}$ | $A_{\rho} \hat{\boldsymbol{\rho}}+A_{\phi} \hat{\boldsymbol{\phi}}+A_{z} \hat{z}$ | $A_{r} \hat{\boldsymbol{r}}+A_{\theta} \hat{\boldsymbol{\theta}}+A_{\phi} \hat{\boldsymbol{\phi}}$ |
| Gradient $\nabla f$ | $\frac{\partial f}{\partial x} \hat{\mathbf{x}}+\frac{\partial f}{\partial y} \hat{\mathbf{y}}+\frac{\partial f}{\partial z} \hat{\mathbf{z}}$ | $\frac{\partial f}{\partial \rho} \hat{\boldsymbol{\rho}}+\frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}}+\frac{\partial f}{\partial z} \hat{\boldsymbol{z}}$ | $\frac{\partial f}{\partial r} \hat{\boldsymbol{r}}+\frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}}+\frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}}$ |
| Divergence $\nabla \cdot \mathbf{A}$ | $\frac{\partial A_{x}}{\partial x}+\frac{\partial A_{y}}{\partial y}+\frac{\partial A_{z}}{\partial z}$ | $\frac{1}{\rho} \frac{\partial\left(\rho A_{\rho}\right)}{\partial \rho}+\frac{1}{\rho} \frac{\partial A_{\phi}}{\partial \phi}+\frac{\partial A_{z}}{\partial z}$ | $\frac{1}{r^{2}} \frac{\partial\left(r^{2} A_{r}\right)}{\partial r}+\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}\left(A_{\theta} \sin \theta\right)+\frac{1}{r \sin \theta} \frac{\partial A_{\phi}}{\partial \phi}$ |
| $\mathbf{C u r l} \nabla \times \mathbf{A}$ | $\left\{\begin{array}{l} \left(\frac{\partial A_{z}}{\partial y}-\frac{\partial A_{y}}{\partial z}\right) \hat{\mathbf{x}}+ \\ \left(\frac{\partial A_{x}}{\partial z}-\frac{\partial A_{z}}{\partial x}\right) \hat{\mathbf{y}}+ \\ \left(\frac{\partial A_{y}}{\partial x}-\frac{\partial A_{x}}{\partial y}\right) \hat{\mathbf{z}} \end{array}\right.$ | $\begin{gathered} \left(\frac{1}{\rho} \frac{\partial A_{z}}{\partial \phi}-\frac{\partial A_{\phi}}{\partial z}\right) \hat{\boldsymbol{\rho}}+ \\ \left(\frac{\partial A_{\rho}}{\partial z}-\frac{\partial A_{z}}{\partial \rho}\right) \hat{\boldsymbol{\phi}}+ \\ \frac{1}{\rho}\left(\frac{\partial\left(\rho A_{\phi}\right)}{\partial \rho}-\frac{\partial A_{\rho}}{\partial \phi}\right) \hat{\boldsymbol{z}} \end{gathered}$ | $\begin{gathered} \frac{1}{r \sin \theta}\left(\frac{\partial}{\partial \theta}\left(A_{\phi} \sin \theta\right)-\frac{\partial A_{\theta}}{\partial \phi}\right) \hat{\boldsymbol{r}}+ \\ \frac{1}{r}\left(\frac{1}{\sin \theta} \frac{\partial A_{r}}{\partial \phi}-\frac{\partial}{\partial r}\left(r A_{\phi}\right)\right) \hat{\boldsymbol{\theta}}+ \\ \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r A_{\theta}\right)-\frac{\partial A_{r}}{\partial \theta}\right) \hat{\boldsymbol{\phi}} \end{gathered}$ |
| Laplace operator $\Delta f=\nabla^{2} f$ | $\frac{\partial^{2} f}{\partial x^{2}}+\frac{\partial^{2} f}{\partial y^{2}}+\frac{\partial^{2} f}{\partial z^{2}}$ | $\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right)+\frac{1}{\rho^{2}} \frac{\partial^{2} f}{\partial \phi^{2}}+\frac{\partial^{2} f}{\partial z^{2}}$ | $\begin{aligned} & \frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial f}{\partial r}\right)+\frac{1}{r^{2} \sin \theta} \frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial f}{\partial \theta}\right)+\frac{1}{r^{2} \sin ^{2} \theta} \frac{\partial^{2} f}{\partial \phi^{2}} \\ & \text { or } \frac{1}{r} \frac{\partial^{2}}{\partial r^{2}}(r f)+\frac{1}{r^{2} \sin \theta} \frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial f}{\partial \theta}\right)+\frac{1}{r^{2} \sin ^{2} \theta} \frac{\partial^{2} f}{\partial \phi^{2}} \end{aligned}$ |
| $\Delta \mathbf{A}=\nabla^{2} \mathbf{A}$ | $\Delta A_{x} \hat{\mathbf{x}}+\Delta A_{y} \hat{\mathbf{y}}+\Delta A_{z} \hat{\mathbf{z}}$ | $\begin{gathered} \left(\Delta A_{\rho}-\frac{A_{\rho}}{\rho^{2}}-\frac{2}{\rho^{2}} \frac{\partial A_{\phi}}{\partial \phi}\right) \hat{\boldsymbol{\rho}}+ \\ \left(\Delta A_{\phi}-\frac{A_{\phi}}{\rho^{2}}+\frac{2}{\rho^{2}} \frac{\partial A_{\rho}}{\partial \phi}\right) \hat{\boldsymbol{\phi}}+ \\ \left(\Delta A_{z}\right) \hat{\boldsymbol{z}} \end{gathered}$ | $\begin{gathered} \left(\Delta A_{r}-\frac{2 A_{r}}{r^{2}}-\frac{2}{r^{2} \sin \theta} \frac{\partial\left(A_{\theta} \sin \theta\right)}{\partial \theta}-\frac{2}{r^{2} \sin \theta} \frac{\partial A_{\phi}}{\partial \phi}\right) \hat{\boldsymbol{r}} \end{gathered}+$ |
| Differential displacement | $d \mathbf{l}=d x \hat{\mathbf{x}}+d y \hat{\mathbf{y}}+d z \hat{\mathbf{z}}$ | $d \mathbf{l}=d \rho \hat{\boldsymbol{\rho}}+\rho d \phi \hat{\boldsymbol{\phi}}+d z \hat{\boldsymbol{z}}$ | $d \mathbf{l}=d r \hat{\mathbf{r}}+r d \theta \hat{\boldsymbol{\theta}}+r \sin \theta d \phi \hat{\boldsymbol{\phi}}$ |
| Differential normal area | $\begin{aligned} d \mathbf{S}= & d y d z \hat{\mathbf{x}}+ \\ & d x d z \hat{\mathbf{y}}+ \\ & d x d y \hat{\mathbf{z}} \end{aligned}$ | $\begin{array}{r} d \mathbf{S}=\rho d \phi d z \hat{\boldsymbol{\rho}}+ \\ d \rho d z \hat{\boldsymbol{\phi}}+ \\ \rho d \rho d \phi \hat{\mathbf{z}} \end{array}$ | $\begin{gathered} d \mathbf{S}=r^{2} \sin \theta d \theta d \phi \hat{\mathbf{r}}+ \\ r \sin \theta d r d \phi \hat{\boldsymbol{\theta}}+ \\ r d r d \theta \hat{\boldsymbol{\phi}} \end{gathered}$ |
| Differential volume | $d v=d x d y d z$ | $d v=\rho d \rho d \phi d z$ | $d v=r^{2} \sin \theta d r d \theta d \phi$ |

## Non-trivial calculation rules:

1. div grad $f=\nabla \cdot(\nabla f)=\nabla^{2} f=\Delta f$ (Laplacian)
2. curl grad $f=\nabla \times(\nabla f)=0$
3. $\operatorname{div} \operatorname{curl} \mathbf{A}=\nabla \cdot(\nabla \times \mathbf{A})=0$
4. curl curl $\mathbf{A}=\nabla \times(\nabla \times \mathbf{A})=\nabla(\nabla \cdot \mathbf{A})-\nabla^{2} \mathbf{A}$ (using Lagrange's formula for the cross product)
5. $\Delta f g=f \Delta g+2 \nabla f \cdot \nabla g+g \Delta f$

## Remarks

- This page uses standard physics notation; some (American mathematics) sources define $\varphi$ as the angle from the $z$-axis instead of $\theta$.
- The function $\operatorname{atan} 2(y, x)$ is used instead of the mathematical function $\arctan (y / x)$ due to its domain and image. The classical $\arctan (y / x)$ has an image of $(-\pi / 2$, $+\pi / 2)$, whereas $\operatorname{atan} 2(y, x)$ is defined to have an image of $(-\pi, \pi]$.


## See also

- Orthogonal coordinates
- Curvilinear coordinates
- Vector fields in cylindrical and spherical coordinates

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Categories: Vector calculus | Coordinate systems

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