Additional Final Study Problems

1) Consider a square wave guide designed to propagate electromagnetic waves along the z-axis. The cross section of the wave guide is a square of side *a*. The magnetic field inside the wave guide only oscillates along the $\pm\hat{y}$ direction.

Recall you can easily find $\vec{E}$ from $\vec{B}$ using $\vec{\nabla}\times\vec{B} = \frac{1}{c^2}\partial_t\vec{E}$.

(a) Write down the general (wave guide) form for B_y which is consistent with the perfect conductor $\vec{B}$ and $\vec{E}$ boundary conditions. Recall the wave guide solutions depend on two integers: m (related to the x dependence), and n (related to the y dependence) . Let B_0 be the amplitude of the B_y oscillation.

ans: $\vec{B} = \hat{y}B_0 \cos\frac{m\pi x}{a}\sin\frac{n\pi y}{a}\ \tilde{e}$

(b) Find the m and n values corresponding to the B_y with the lowest ω_{cutoff} . Find this cut-off frequency in terms of *a.*

Assume we are in the lowest cut-off mode for what follows and answer all questions in terms of in terms of k, ω, a, B_0 and physical constants.

(c) Find expressions for the surface current $\vec{K}$ on each of the four walls

(d) Find the charge density (σ) on each of the four walls

(e) Show your $\vec{K}$ and σ satisfy $\vec{\nabla}\bullet\vec{K} + \partial_t\sigma = 0$ on each of the four walls.

(f) Calculate the time averaged power carried in the wave guide. Is any power carried at the cut-off frequency? ans: $\langle \mathcal{P} \rangle = \dfrac{kc^2B_0^2a^2}{4\mu_0\omega}$

2) Consider a surface (such as a sphere or cylinder) with a local surface current plane which creates a discontinuity for a static B-field in the $\hat{z}$ direction. Let's put the normal of the local current plane in the $\hat{x}$ direction, the direction of the local surface current in the $\hat{y}$ direction, and the center of the local current plane at $x = y = z = 0$.

(a) Find the pressure acting in the $\hat{x}$ direction due to the surface current in terms of the z component of the magnetic field just below (${}^{-}B_z$) and just above (${}^{+}B_z$) the current plane using the Maxwell stress tensor. What is the bounding surface that you must use to compute this pressure?

$$\boxed{\text{ans: press}_x = -\frac{{}^{+}B_z^2 - {}^{-}B_z^2}{2\mu_0}}$$

(b) Show that the pressure you computed in (a) is given by $\hat{x} \bullet \vec{K} \times \vec{B}_{\text{ave}}$ where $\vec{B}_{\text{ave}} = \frac{{}^{+}B_z + {}^{-}B_z}{2}\hat{z}$. Write $\vec{K}$ in terms of ${}^{+}B_z$ and ${}^{-}B_z$. $\boxed{\text{ans:} \vec{K} = -\frac{\left({}^{+}B_z - {}^{-}B_z\right)\hat{y}}{\mu_0}}$

(c) Check your results for (a) and (b) by considering (1) the pressure on a long solenoid of radius R carrying providing $\vec{B}(s < R) = B_0\hat{z}$ and (2) the pressure on an infinite current plane with *x=0* and $\vec{K} = K\hat{y}$.

3) A particle has an energy of $\mathcal{E}$ and a velocity of $c\vec{\beta}$ in the lab frame.

(a) Find $\mathcal{E}'$ which is the energy of the particle in a prime frame traveling with a lab frame velocity of $\vec{v}$ terms of $\mathcal{E}$, $\vec{\beta}$, $\vec{v}$ and $\gamma = 1/\sqrt{1-\vec{v} \bullet \vec{v}/c^2}$. $\boxed{\text{ans:} \mathcal{E}' = \gamma\mathcal{E}\left(1 - \frac{\vec{\beta} \bullet \vec{v}}{c}\right)}$

(b) Find $\mathcal{E}'$ for the case where $\vec{v} = c\vec{\beta}$. Do you understand your answer?